Chapter 2

Unitary symmetry and quarks

2.1 Eightfold way. Mass formulae in $SU(3)$.

2.1.1 Baryon and meson unitary multiplets

Let us return to baryons $1/2^+$ and mesons 0^-. As we remember there are 8 particles in each class: 8 baryons: - isodublets of nucleon (proton and neutron) and cascade hyperons $\Xi^{0,-}$, isotriplet of Σ-hyperons and isosinglet Λ, and 8 mesons: isotriplet π, two isodublets of strange K-mesons and isosinglet η. Let us try to write baryons $B(1/2^+)$ as a 8-vector of real fields $B = (B_1, ..., B_8)$= $(\vec{\Sigma}, N_4, N_5, B_6, B_7, B_8)$, where $\vec{\Sigma}$=(B_1, B_2, B_3)= $(\Sigma_1, \Sigma_2, \Sigma_3)$. Then the basis vector of the 8-dimensional representation could be written in the matrix form:

$$B^{\alpha}_{\beta} = \frac{1}{\sqrt{2}} \sum\nolimits_{k=1}^{8} \lambda_k B_k =$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \Sigma_3 + \frac{1}{\sqrt{3}} B_8 & \Sigma_1 - i\Sigma_2 & N_4 - iN_5 \\ \Sigma_1 + i\Sigma_2 & -\Sigma_3 + \frac{1}{\sqrt{3}} B_8 & B_6 - iB_7 \\ N_4 + iN_5 & B_6 + iB_7 & -\frac{2}{\sqrt{3}} B_8 \end{pmatrix} = \tag{2.1}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}.$$

At the left upper angle of the matrix we see the previous espression (1.23) from theory of isotopic group $SU(2)$. In a similar way pseudoscalar mesons yield 3×3 matrix

$$P^\alpha_\beta = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{2.2}$$

Thus one can see that the classiification proves to be very impressive: instead of 16 particular particles we have now only 2 unitary multiplets. But what corollaries would be? The most important one is a deduction of mass formulae, that is, for the first time it has been succeded in relating among themselves of the masses of different elementary particles of the same spin.

2.1.2 Mass formulae for octet of pseudoscalar mesons

As it is known, mass term in the Lagrangian for the pseudoscalar meson field described by the wave function P has the form quadratic in mass (to assure that Lagrange-Euler equation for the free point-like meson would yield Gordon equation where meson mass enters quadratically)

$$L^P_m = m^2_P P^2,$$

and for octet of such mesons with all the masses equal (degenerated):

$$L^P_m = m^2_P P^\alpha_\beta P^\beta_\alpha$$

(note that over repeated indices there is a sum), while $m_\pi = 140$ MeV,$m_K =$ 490 MeV, $m_\eta = 548$ MeV. Gell-Mann proposed to refute principle that Lagrangian should be scalar of the symmetry group of strong intereractions, here unitary group $SU(3)$, and instead introduce symmetry breaking but in such a way as to conserve isotopic spin and strangeness (or hypercharge $Y = S + B$ where B is the baryon charge equal to zero for mesons). For this purpose the symmetry breaking term should have zero values of isospin and hypercharge. Gell-Mann proposed a simple solution of the problem, that is, the mass term should transform as the 33-component of the octet formed by product of two meson octets. (Note that in either meson or baryon octet

33-component of the matrix has zero values of isospin and hypercharge) First it is necessary to extract octet from the product of two octets entering Lagrangian. It is natural to proceed contracting the product $P^{\alpha}_{\eta}P^{\beta}_{\gamma}$ along upper and sub indices as $P^{\alpha}_{\gamma}P^{\gamma}_{\beta}$ or $P^{\gamma}_{\beta}P^{\alpha}_{\gamma}$ and subtract the trace to obtain regular octets

$$M^{\alpha}_{\beta} = P^{\alpha}_{\gamma}P^{\gamma}_{\beta} - \frac{1}{3}P^{\eta}_{\gamma}P^{\gamma}_{\eta}$$

,

$$N^{\alpha}_{\beta} = P^{\gamma}_{\beta}P^{\alpha}_{\gamma} - \frac{1}{3}P^{\eta}_{\gamma}P^{\gamma}_{\eta}$$

$M^{\alpha}_{\alpha} = 0, \quad N^{\alpha}_{\alpha} = 0$ (over repeated indices there is a sum). Components 33 of the octets M^{3}_{3} и N^{3}_{3} would serve us as terms which break symmetry in the mass part of the Lagrangian L^{P}_{m}. One should only take into account that in the meson octet there are both particles and antiparticles. Therefore in order to assure equal masses for particles and antiparticles, both symmetry breaking terms should enter Lagrangian with qual coefficients. As a result mass term of the Lagrangian can be written in the form

$$L^{P}_{m} = m^{2}_{P}P^{\alpha}_{\beta}P^{\beta}_{\alpha} + m^{2}_{1P}(M^{3}_{3} + N^{3}_{3}) =$$

$$= m^{2}_{0P}P^{\alpha}_{\beta}P^{\beta}_{\alpha} + m^{2}_{1P}(P^{3}_{\beta}P^{\beta}_{3} + P^{\alpha}_{3}P^{3}_{\alpha}).$$

Taking together coefficients in front of similar bilinear combinations of the pseudoscalar fields we obtain

$$m^{2}_{\pi} = m^{2}_{0P}, \quad m^{2}_{K} = m^{2}_{0P} + m^{2}_{1P}, \quad m^{2}_{\eta} = m^{2}_{0P} + \frac{4}{3}m^{2}_{1P},$$

wherefrom the relation follows immediately

$$4m^{2}_{K} = 3m^{2}_{\eta} + m^{2}_{\pi}, \quad 4 \times 0,245 = 3 \times 0,30 + 0,02(\textit{Г}\textit{э}\textit{В})^{2}.$$

The agreement proves to be impressive taking into account clearness and simplicity of the formalism used.

2.1.3 Mass formulae for the baryon octet $J^{P} = \frac{1}{2}^{+}$

Mass term of a baryon B с $J^{P} = \frac{1}{2}^{+}$ in the Lagrangian is usually linear in mass (to assure that Lagrange-Euler equation of the full Lagrangian for free point-like baryon would be Dirac equation where baryon mass enter linearly)

$$L_m^B = m_B \bar{B} B.$$

For the baryon octet B_α^β with the degenerated (all equal) masses the corresponding part of the Lagrangian yields

$$L_m^B = m_B \bar{B}_\beta^\alpha B_\alpha^\beta,$$

But real masse are not degenerated at all: $m_N \sim 940, \quad m_\Sigma \sim 1192, \quad m_\Lambda \sim 1115 \quad m_\Xi \sim 1320$ (in MeV). Also here Gell-Mann proposed to introduce mass breaking through breaking in a definite way a symmetry of the Lagrangian:

$$L_m^B = m_0 \bar{B}_\beta^\alpha B_\alpha^\beta + m_1 \bar{B}_\beta^3 B_3^\beta + m_2 \bar{B}_3^\alpha B_\alpha^3).$$

Note that here there are two terms with the 33-component as generally speaking $m_1 \neq m_2$. (While mesons and antimesons are in the same octet, baryons and antibaryons forms two different octets) Then for particular baryons we have:

$$p = B_3^1, \quad n = B_3^2 \quad m_p = m_n = m_0 + m_1$$

$$\Sigma^+ = B_2^1, \quad \Sigma^- = B_1^2 \quad m_{\Sigma^{\pm,0}} = m_0$$

$$-\frac{2}{\sqrt{6}}\Lambda_0 = B_3^3, \quad m_\Lambda = m_0 + \frac{2}{3}(m_1 + m_2),$$

$$\Xi_- = B_1^3, \quad \Xi^0 = B_2^3 \quad m_{\Xi^{-,0}} = m_0 + m_2$$

The famous Gell-Mann-Okubo mass relation follows immediately:

$$2(m_N + m_\Xi) = m_\Sigma + 3m_\Lambda, \quad 4520 = 4535.$$

(Values at the left-hand side (LHS) and right-hand side (RHS) are given in MeV.) The agreement with experiment is outstanding which has been a stimul to further application of the unitary Lie groups in particle physics.

2.1.4 Nonet of the vector meson and mass formulae

Mass formula for the vector meson is the same as that for pseudoscalar ones (in this model unitary space do not depends on spin indices).

But number of vector mesons instead of 8 is 9, therefore we apply this formula taking for granted that it is valid here, to find mass of the isoscalar vector meson ω_0 of the octet:

$$m_{\omega_0}^2 = \frac{1}{3}(4m_{K^*}^2 - m_\rho^2),$$

wherefrom $m_{\omega_0} = 930$ MeV. But there is no such isoscalar vector meson of this mass. Instead there are a meson ω with the mass $m_\omega = 783$ MeV and a meson ϕ with the mass $m_\phi = 1020$ Mev. Okubo was forced to introduce nonet of vector mesons as a direct sum of the octet and the singlet

$$V_\beta^\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{6}}\omega_8 & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} + \\ + \begin{pmatrix} \frac{1}{\sqrt{3}}\phi_0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}}\phi_0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}}\phi_0 \end{pmatrix} \tag{2.3}$$

which we write going a little forward as

$$V_\beta^\alpha = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{2.4}$$

$$\begin{pmatrix} \omega \\ \phi \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\ -\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} \end{pmatrix} \begin{pmatrix} \omega_8 \\ \phi_0 \end{pmatrix},$$

where $\sqrt{\frac{1}{3}}$ is essentially $cos\theta_{\omega\phi}$, $\theta_{\omega\phi}$ being the angle of ideal mixing of the octet and singlet states with $I = 0, \quad S = 0$. Let us stress once more that introduction of this angle was caused by discrepancy of the mass formula for vector mesons with experiment.

"Nowadays (1969!) there is no serious theoretical basis to treat V_9 (vector nonet–V.Z.) as to main quantity never extracting from it ω_0 (our ϕ_0–V.Z.) as SpV_9 so Okubo assertion should be seen as curious but not very profound observation." S.Gasiorowicz, "Elementary particles physics", John Wiley & Sons, Inc. NY-London-Sydney, 1968.

We will see a little further that the Okubo assertion is not only curious but also very profound.

2.1.5 Decuplet of baryon resonances with $J^P = \frac{3}{2}^+$ and its mass formulae

Up to 1963 nine baryon resonances with $J^P = \frac{3}{2}^+$ were established: isoquartet (with $I = \frac{3}{2}$) $\Delta(1232) \to N+\pi$; isotriplet $\Sigma^*(1385) \to \Lambda+\pi, \Sigma+\pi$; isodublet $\Xi^*(1520) \to \Xi+\pi$. But in $SU(3)$ there is representation of the dimension 10, an analogue to IR of the dimension 4 in $SU(2)$ (with $I = \frac{3}{2}$) which is given by symmetric tensor of the 3rd rank (what with symmetric tensor describing the spin state $J^P = \frac{3}{2}^+$ yields symmetric wave function for the particle of half-integer spin!!!). It was absent a state with strangeness -3 which we denote as Ω. With this state decuplet can be written as:

$$\begin{array}{ccccccc}
\Delta^-_{222} & & \Delta^0_{221} & & \Delta^+_{211} & & \Delta^{++}_{111} \\
& \Sigma^{*-}_{223} & & \Sigma^{*0}_{123} & & \Sigma^{*+}_{113} & \\
& & \Xi^{*-}_{233} & & \Xi^{*0}_{133} & & \\
& & ? & \Omega^-_{333} & ? & &
\end{array}$$

Mass term of the Lagrangian for resonance decuplet $B^{*\alpha\beta\gamma}$ following Gell-Mann hypothesis about octet character of symmetry breaking of the Lagrangian can be written in a rather simple way:

$$L^{B^*}_{M^*} = M^*_0 \bar{B}^*_{\alpha\beta\gamma} B^{\alpha\beta\gamma} + M^*_1 \bar{B}^*_{3\beta\gamma} B^{3\beta\gamma}.$$

Really from unitary wave functions of the decuplet of baryon resonances $B^{*\alpha\beta\gamma}$ and corresponding antidecuplet $\bar{B}_{*\alpha\beta\gamma}$ due to symmetry of indices it is possible to constract an octet in a unique way. The result is:

$$M_\Delta = M^*_0$$

$$\begin{aligned}
M_{\Sigma^*} &= M^*_0 + M^*_1 \\
M_{\Xi^*} &= M^*_0 + 2M^*_1 \\
M_{\Omega^-} &= M^*_0 + 3M^*_1
\end{aligned}$$

Mass formula of this kind is named equidistant. It is valid with sufficient accuracy, the step in mass scale being around 145 MeV. But in this case the predicted state of strangeness -3 and mass $(1530 + 145) = 1675$ MeV cannot be a resonance as the lighest two-particle state of strangeness -3 would be $(\Xi(1320)K(490))$ with the mass 1810 MeV! It means that if it exists it should be a particle stable relative to strong interactions and should decay through weak interactions in a cascade way loosing strangeness -1 at each step.

This prediction is based entirely on the octet symmetry breaking of the Lagrangian mass term of the baryon decuplet $B^{*\alpha\beta\gamma}$.

Particle with strangeness -3 Ω^- was found in 1964, its mass turned out to be $(1672, 5 \pm 0, 3)$ MeV coinciding exactly with $SU(3)$ prediction!

It was a triumph of unitary symmetry! The most of physicists believed in it from 1964 on. (By the way the spin of the Ω^- hyperon presumably equal to 3/2 has never been measured.)

2.2 Praparticles and hypothesis of quarks

Upon comparing isotopic and unitary symmetry of elementary particles one can note that in the case of isotopic symmetry the lowest possible IR of the dimension 2 is often realized which has the basis $(1 \quad 0)^T$, $(0 \quad 1)^T$; along this representation, for example, N, Ξ, K, Ξ^*, K^* transform, however at the same time unitary multiplets of hadrons begin from the octet (analogue of isotriplet in $SU(2)_I$).

The problem arises whether in nature the lowest spinor representations are realized. In other words whether more elementary particles exist than hadrons discussed above?

For methodical reasons let us return into the times when people was living in caves, used telegraph and vapor locomotives and thought that π-mesons were bounded states of nucleons and antinucleons and try to understand in what way one can describe these states in isotopic space.

Let us make a product of spinors $N^a, \bar{N}_b, a, b = 1, 2$, and then subrtract and add the trace $\bar{N}_c N^c, c = 1, 2, 3$, expanding in this way a product of two irreducible representations (IR) (two spinors) into the sum of IR's:

$$\bar{N}_b \times N^a = (\bar{N}_b N^a - \frac{1}{2}\delta^a_b \bar{N}_c N^c) + \frac{1}{2}\delta^a_b \bar{N}_c N^c, \tag{2.5}$$

what corresponds to the expansion in terms of isospin $\frac{1}{2} \times \frac{1}{2} = 1 + 0$, or (in terms of IR dimensions) $2 \times 2 = 3 + 1$. In matrix form

$$(\bar{p}, \bar{n}) \times \begin{pmatrix} p \\ n \end{pmatrix} = \begin{pmatrix} (\bar{p}p - \bar{n}n)/2 & \bar{n}p \\ \bar{p}n & -(\bar{p}p - \bar{n}n)/2 \end{pmatrix} +$$
$$+ \begin{pmatrix} (\bar{p}p + \bar{n}n)/2 & 0 \\ 0 & (\bar{p}p + \bar{n}n)/2 \end{pmatrix}, \tag{2.6}$$

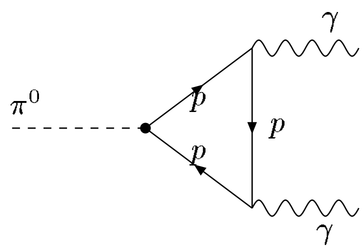
which we identify for the J=0 state of nucleon and antinucleon spins and zero orbital angular momentum with the pion isotriplet and isosinglet η:

$$\pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}\eta^0 & 0 \\ 0 & \frac{1}{\sqrt{2}}\eta^0 \end{pmatrix},$$

while for for the J=1 state of nucleon and antinucleon spins and zero orbital angular momentum with the ρ-meson isotriplet and isosinglet ω:

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 & \rho^+ \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 \end{pmatrix} + \begin{pmatrix} \frac{1}{\sqrt{2}}\omega^0 & 0 \\ 0 & \frac{1}{\sqrt{2}}\omega^0 \end{pmatrix}.$$

This hypothesis was successfully used many times. For example, within this hypothesis the decay π^0 into two γ-quanta was calculated via nucleon loop,



The answer coincided exactly with experiment what was an astonishing achievement. Really, mass of two nucleons were so terribly larger than the mass of the π^0-meson that it should be an enormous bounding energy between nucleons. However the answer was obtained within assumption of quasi-free nucleons (1949, one of the achievements of Feynman diagram technique applied to hadron decays.)

With observation of hyperons number of fundamental baryons was increased suddenly. So first composite models arrived. Very close to model of unitary symmetry was Sakata model with proton, neutron and Λ hyperon as a fundamental triplet. But one was not able to put baryon octet into such model. However an idea to put something into triplet remains very attractive.

2.3 Quark model. Mesons in quark model.

Model of quarks was absolutely revolutionary. Gell-Mann and Zweig in 1964 assumed that there exist some praparticles transforming along spinor representation of the dimension 3 of the group $SU(3)$ (and correspondingly antipraparticles transforming along conjugated spinor representation of the dimension 3), and all the hadrons are formed from these fundamental particles. These praparticles named quarks should be fermions (in order to form existing baryons), and let it be fermions with $J^P = \frac{1}{2}^+$ $q^\alpha, \alpha = 1, 2, 3, q^1 = u, q^2 = d, q^3 = s$. Note that because one needs at least three quarks in order to form baryon of spin 1/2, electric charge as well as hypercharge turn out to be дробными non-integer(!!!) which presented 40 years ago as open heresy and for many of us really unacceptable one.

Quarks should have the following quantum numbers:

	Q	I	I_3	Y=S+B	B
u	2/3	1/2	1/2	1/3	1/3
d	-1/3	1/2	-1/2	1/3	1/3
s	-1/3	0	0	-2/3	1/3

in order to assure the right quantum numbers of all 8 known baryons of spin 1/2 (2.1): $p(uud)$, $n(ddu)$, $\Sigma^+(uus)$, $\Sigma^0(uds)$, $\Sigma^-(dds)$, $\Lambda(uds)$, $\Xi^0(ssu)$, $\Xi^-(ssd)$. In more details we shall discuss baryons a little further in another section in order to maintain the continuity of this talk.

First let us discuss meson states. We can try to form meson states out of quarks in complete analogy with our previous discussion on nucleon-antinucleon states and Eqs.(2.5,2.6):

$$\bar{q}_\beta \times q^\alpha = (\bar{q}_\beta q^\alpha - \frac{1}{3}\delta^\alpha_\beta \bar{q}_\gamma q^\gamma) + \frac{1}{3}\delta^\alpha_\beta \bar{q}_\gamma q^\gamma),$$

$$(\bar{u}, \bar{d}, \bar{s}) \times \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix} =$$

$$\begin{pmatrix} D_1 & \bar{d}u & \bar{s}u \\ \bar{u}d & D_2 & \bar{s}d \\ \bar{u}s & \bar{d}s & D_3 \end{pmatrix} + \frac{1}{3}(\bar{u}u + \bar{d}d + \bar{s}s)I, \tag{2.7}$$

where

$$D_1 = \bar{u}u - \frac{1}{3}\bar{q}q = \frac{1}{2}(\bar{u}u - \bar{d}d) + \frac{1}{6}(\bar{u}u + \bar{d}d - 2\bar{s}s) =$$
$$= \frac{1}{2}\bar{q}\lambda_3 q + \frac{1}{2\sqrt{3}}\bar{q}\lambda_8 q,$$
$$D_2 = \bar{d}d - \frac{1}{3}\bar{q}q = -\frac{1}{2}(\bar{u}u - \bar{d}d) + \frac{1}{6}(\bar{u}u + \bar{d}d - 2\bar{s}s) =$$
$$= -\frac{1}{2}\bar{q}\lambda_3 q + \frac{1}{2\sqrt{3}}\bar{q}\lambda_8 q,$$
$$D_3 = \bar{s}s - \frac{1}{3}\bar{q}q = -\frac{2}{6}(\bar{u}u + \bar{d}d - 2\bar{s}s) = -\frac{1}{\sqrt{3}}\bar{q}\lambda_8 q.$$

We see that the traceless matrix obtained here could be identified with the meson octet $J^P = 0^-$(S-state), the quark structure of mesons being:

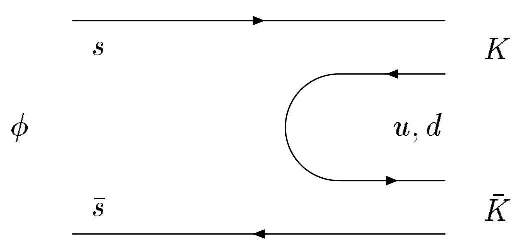
$$\pi^- = (\bar{u}d), \quad \pi^+ = (\bar{d}u),$$
$$K^- = (\bar{u}s), \quad K^+ = (\bar{s}u),$$
$$\pi^0 = \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d),$$
$$K^0 = (\bar{s}d), \quad \bar{K}^0 = (\bar{d}s),$$
$$\eta = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s).$$

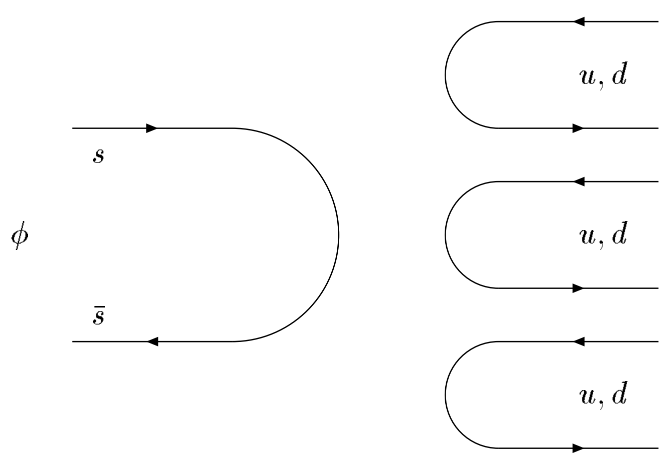
In a similar way nonet of vector mesons Eq.(2.10) can be constructed. But as they are in 9, we take straightforwardly the first expression in Eq.(2.7) with the spins of quarks forming $J = 1$ (always S-state of quarks):

$$(\bar{u}, \bar{d}, \bar{s})_\uparrow \times \begin{pmatrix} u \\ d \\ s \end{pmatrix}_\uparrow = \begin{pmatrix} \bar{u}u & \bar{d}u & \bar{s}u \\ \bar{u}d & \bar{d}d & \bar{s}d \\ \bar{u}s & \bar{d}s & \bar{s}s \end{pmatrix}_{J=1} =$$
$$= \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{2.8}$$

This construction shows immediately a particular structure of the ϕ meson : it contains only strange quarks!!!

Immediately it becomes clear more than strange character of its decay channels. Namely while ω meson decays predominantly into 3 pions, the ϕ meson practically does not decay in this way $(2,5 \pm 09)\%$ although energetically it is very 'profitable' and, instead, decays into the pair of kaons ($(49,1 \pm 0,9)\%$ into the pair K^+K^- and $(34,3 \pm 0,7)\%$ into the pair $K^0_L K^0_S$). This strange experimental fact becomes understandable if we expose quark diagrams at the simplest level:





Thus we have convinced ourselves that Okubo note on nonet was not only curious but also very profound.

We see also that experimental data on mesons seem to support existence of three quarks.

But is it possible to estimate effective masses of quarks? Let us assume that ϕ-meson is just made of two strange quarks, that is, $m_s = m(\phi)/2 \equiv 510$ MeV. An effective mass of two light quarks let us estimate from nucleon mass as $m_u = m_d = M_p/s \equiv 310$ MeV. These masses are called constituent ones. Now let us look how it works.

$$M_{p(uu,d)} = M_{n(dd,u)} = 930 \quad \text{Mev} \qquad \text{(input)}(\sim 940)\text{exp}$$

$$M_{\Sigma(qq,s)} = 1130 \quad \text{Mev} \qquad (\sim 1192)\text{exp}$$

$$M_{\Lambda(uds)} = 1130, \quad \text{Mev} \qquad (\sim 1115)\text{exp}$$

$$M_{\Xi(ss,q)} = 1330 \quad \text{Mev} \qquad (\sim 1320)\text{exp}$$

(There is one more very radical question:

Whether quarks exist at all?

From the very beginning this question has been the object of hot discussions. Initially Gell-Mann seemed to consider quarks as some suitable mathematical object for particle physics. Nowadays it is believed that quarks are as real as any other elementary particles. In more detail we discuss it a little later.)

"Till Parisina's fatal charms
Again attracted every eye..."–

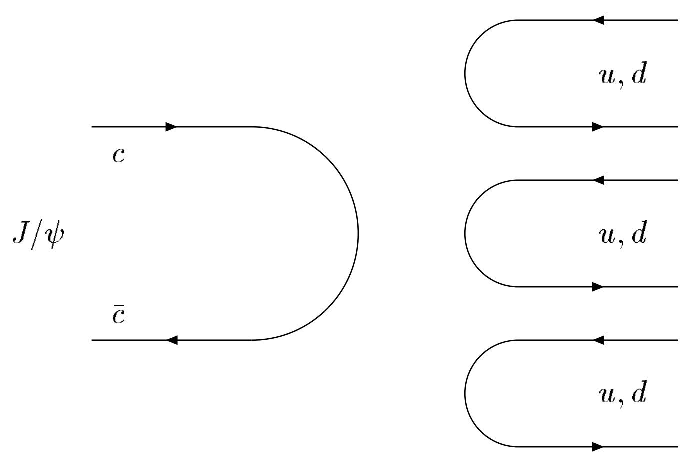
Lord Byron, 'Parisina'

2.4 Charm and its arrival in particle physics.

Thus, there exist three quarks!

During 10 years everybody was thinking in this way. But then something unhappened happened.

In november 1974 on Brookehaven proton accelerator with max proton energy 28 GeV (USA) and at the electron-positron rings SPEAR(SLAC,USA) it was found a new particle– J/ψ vector meson decaying in pions in the hadron channel at surprisingly large mass 3100 MeV and surprisingly long mean life and, correspondingly, small width at the level of 100 KeV although for hadrons characteristic widths oscillate between 150 MeV for rho meson, 8 Mev for ω and 4 MeV for ϕ meson. Taking analogy with suppression of the 3-pion decay of the vector ϕ meson which as assumed is mainly $(\bar{s}s)$, state the conclusion was done that the most simple solution would be hypothesis of existence of the 4th quark with the new quantum number "charm". In this case $J/\psi(3100)$ is the $(\bar{c}c)$ vector state with hidden charm.

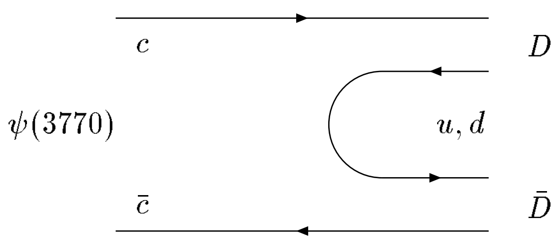


And what is the mass of charm quark? Let us make a bold assumption that as in the case of the $\phi(1020)$ meson where mass of a meson is just double value of the ('constituent') strange quark mass (500 MeV) (so called 'constituent' masses of quarks u and d are around 300 Mev as we have see) mass of the charm quark is just half of that of the $J/\psi(3100)$ particle that is around 1500 MeV (more than 1.5 times proton mass!).

But introduction of a new quark is not so innocue. One assumes with this hypothesis existence of the whole family of new particles as mesons with the charm with quark content $(\bar{u}c)$, $(\bar{c}u)$, $(\bar{d}c)$, $(\bar{c}d)$, with masses around (1500+300=1800) MeV at least and also $(\bar{s}c)$ и $(\bar{c}s)$ with masses around (1500+500=2000) MeV. As it is naturally to assume that charm is conserved in strong interaction as strangeness does, these mesons should decay due to weak interaction loosing their charm. For simplicity we assume that masses of these mesons are just sums of the corresponding quark masses.

Let us once more use analogy with the vector $\phi(1020)$ meson main decay channel of which is the decay $K(490)$ $\bar{K}(490)$ and the production of this meson with the following decay due to this channel dominates processes at the electron-positron rings at the total energy of 1020 MeV.

If analogue of $J/\psi(3100)$ with larger mass exists, namely, in the mass region $2\times(1500+300)=3600$ MeV, in this case such meson should decay mostly to pairs of charm mesons. But such vector meson $\psi(3770)$ was really found at the mass 3770 MeV and the main decay channel of it is the decay to two new particles– pairs of charm mesons $D^0(1865)\bar{D}^0(1865)$ or $D^+(1870)D^-(1870)$ and the corresponding width is more than 20 MeV!!!



New-found charm mesons decay as it was expected due to weak interaction what is seen from a characteristic mean life at the level of $10^{-12} - 10^{-13}$ s.

Unitary symmetry group for particle classification grows to $SU(4)$. It is to note that it would be hardly possible to use it to construct mass formulae as $SU(3)$ because masses are too different in 4-quark model. In any case a problem needs a study.

In the model of 4 quarks (4 flavors as is said today) mesons would transform along representations of the group $SU(4)$ contained in the direct product of the 4-dimensional spinors 4 and $\bar{4}$,: $\bar{4}\times 4 = 15+1$, or

$$\bar{q}_\beta \times q^\alpha = (\bar{q}_\beta q^\alpha - \frac{1}{4}\delta^\alpha_\beta \bar{q}_\gamma q^\gamma) + \frac{1}{4}\delta^\alpha_\beta \bar{q}_\gamma q^\gamma),$$

where now $\alpha, \beta, \gamma = 1, 2, 3, 4$.

$$P^\alpha_\beta = \begin{pmatrix} \eta(2980) & D^0(1865) & D^+(1870) & F^+(1969) \\ D^0(1865) & \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ D^-(1870) & \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ F^-(1969) & K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} \tag{2.9}$$

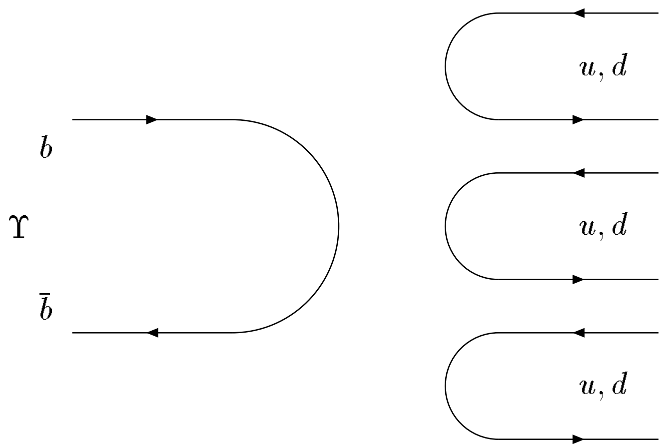
$$V^\alpha_\beta = \begin{pmatrix} \psi(3100) & D^{*0}(2007) & D^{*+}(2010) & F^{*+}(2112) \\ D^{*0}(2007) & \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^+ \\ D^{*-}(2010) & \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ F^{*-}(2112) & K^{-*} & \bar{K}^{*0} & \phi \end{pmatrix} \tag{2.10}$$

2.5 The fifth quark. Beauty.

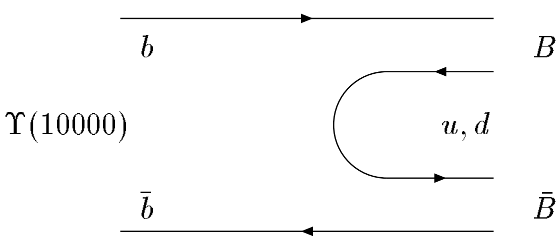
Per la bellezza delle donne
molti sono periti... Eccl.Sacra Bibbia

Because of beauty of women
many men have perished... Eccl. Bible

Victorious trend for simplicity became even more clear after another important discouvery: when in 1977 a narrow vector resonance was found at the mass around 10 GeV, $\Upsilon(1S)(9460)$,$\Gamma \approx 50$Kev, everybody decided that there was nothing to think about, it should be just state of two new, 5th quarks of the type $\bar{s}s$ or $\bar{c}c$. This new quark was named b from *beauty* or *bottom* and was ascribed the mass around 5 GeV $\sim M_\Upsilon/2$, a half of the mass of a new meson with the hidden "beauty" $\Upsilon(1S)(9460) = (\bar{b}b)$.



Immediately searches for next excited states was begun which should similarly to $\psi(3770)$ have had essentially wider width and decay into mesons with the quantum number of "beauty". Surely it was found. It was $\Upsilon(4S)(10580)$,$\Gamma \approx$ 10 MeV. It decays almost fully to meson pair $\bar{B}B$ and these mesons B have mass $\approx 5300 = (5000 + 300)$ MeVB and mean life around 1.5 ns.



2.6 Truth or top?

Truth is not to be sought in the good fortune of any particular conjuncture of time, which is uncertain, but in the light of nature and experience, which is eternal. Francis Bacon

But it was not all the story as to this time there were already 6 leptons $e^-, \nu_e, \mu^-, \nu_\mu$, τ^-, ν_τ (and 6 antileptons) and only 5 quarks: c, u with the electric charge +2/3e and d, s, b with the electric charge -1/3e. Leaving apart theoretical foundations (though they are and serious) we see that a symmetry between quarks and leptons and between the quarks of different charge is broken. Does it mean that there exist one more, 6th quark with the new 'flavour' named "truth" or "top" ?

For 20 years its existence was taking for granted by almost all the physicists although there were also many attempts to construct models without the 6th quark. Only in 1996 t-quark was discovered at the mass close to the nucleus of ${}^{71}Lu^{175}$, $m_t \approx 175$GeV. We have up to now rather little to say about it and particles containing t-quark but the assertion that t-quark seems to be uncapable to form particles.

2.7 Baryons in quark model

Up to now we have considered in some details mesonic states in frameworks of unitary symmetry model and quark model up to 5 quark flavours. Let us return now to $SU(3)$ and 3-flavour quark model with quarks $q^1 = u, q^2 = d, q^3 = s$ and let us construct baryons in this model. One needs at least three quarks to form baryons so let us make a product of three 3-spinors of $SU(3)$ and search for octet in the expansion of the triple product of the IR's: $3 \times 3 \times 3 = 10 + 8 + 8' + 1$. As it could be seen it is even two octet IR's in this product so we can proceed to construct baryon octet of quarks. Expanding of the product of three 3-spinors into the sum of IR's is more complicate than for the meson case. We should symmetrize and antisymmetrize all indices to get the result:

$$
\begin{aligned}
&q^{\alpha}\times q^{\beta}\times q^{\gamma}=\frac{1}{6}(q^{\alpha}q^{\beta}q^{\gamma}+q^{\beta}q^{\alpha}q^{\gamma}+q^{\alpha}q^{\gamma}q^{\beta}+q^{\beta}q^{\gamma}q^{\alpha}+q^{\gamma}q^{\beta}q^{\alpha}+q^{\gamma}q^{\alpha}q^{\beta})+\\
&+\frac{1}{6}(2q^{\alpha}q^{\beta}q^{\gamma}-2q^{\beta}q^{\alpha}q^{\gamma}+q^{\alpha}q^{\gamma}q^{\beta}-q^{\beta}q^{\gamma}q^{\alpha}+q^{\gamma}q^{\beta}q^{\alpha}-q^{\gamma}q^{\alpha}q^{\beta})+\\
&+\frac{1}{6}(2q^{\alpha}q^{\beta}q^{\gamma}+2q^{\beta}q^{\alpha}q^{\gamma}-q^{\alpha}q^{\gamma}q^{\beta}+q^{\beta}q^{\gamma}q^{\alpha}-q^{\gamma}q^{\beta}q^{\alpha}+q^{\gamma}q^{\alpha}q^{\beta})+\\
&+\frac{1}{6}(q^{\alpha}q^{\beta}q^{\gamma}-q^{\beta}q^{\alpha}q^{\gamma}-q^{\alpha}q^{\gamma}q^{\beta}+q^{\beta}q^{\gamma}q^{\alpha}+q^{\gamma}q^{\beta}q^{\alpha}-q^{\gamma}q^{\alpha}q^{\beta})=T^{(\alpha\beta\gamma)}+T^{[\alpha\beta]\gamma}+T^{[\alpha\gamma]\beta}+T^{[\alpha\beta\gamma]}
\end{aligned}
$$

All indices go from 1 to 3. Symmetrical tensor of the 3rd rank has the dimension $N_n^{SSS} = (n^3 + 3n^2 + 2n)/6$ and for n=3 $N_3^{SSS} = 10$. Antisymmetrical tensor of the 3rd rank has the dimension $N_n^{AAA} = (n^3 - 3n^2 + 2n)/6$ and for n=3 $N_3^{AAA} = 1$. Tensors of mixed symmetry of the dimension $N_n^{mix} = n(n^2 - 1)/3$ only for n=3 (N_3^{mix}=8) could be rewritten in a more suitable form as T^{α}_{β} upon using absolutely antisymmetric tensor (or Levi-Civita tensor) of the 3rd rank $\epsilon_{\beta\delta\eta}$ which transforms as the singlet IR of the group $SU(3)$. Really,

$$\epsilon_{\alpha'\beta'\gamma'} = u^{\alpha}_{\alpha'} u^{\beta}_{\beta'} u^{\gamma}_{\gamma'} \epsilon_{\alpha\beta\gamma} :$$

$$
\begin{aligned}
\epsilon_{123} = u^1_1 u^2_2 u^3_3 \epsilon_{123} + u^2_1 u^3_2 u^1_3 \epsilon_{231} + u^3_1 u^1_2 u^2_3 \epsilon_{312} + u^1_1 u^3_2 u^2_3 \epsilon_{132}\\
+ u^3_1 u^2_2 u^1_3 \epsilon_{321} + u^2_1 u^1_2 u^3_3 \epsilon_{213} =
\end{aligned}
$$

$$= (u_1^1 u_2^2 u_3^3 + u_1^2 u_2^3 u_3^1 + u_1^3 u_2^1 u_3^2 - u_1^1 u_2^3 u_3^2 - u_1^3 u_2^2 u_3^1 - u_1^2 u_2^1 u_3^3)\epsilon_{123} =$$

$$= DetU\epsilon_{123} = \epsilon_{123}$$

as $DetU = 1$. (The same for ϵ_{213} etc.)

For example, partly antisymmetric 8-dimensional tensor $T^{[\alpha\beta]\gamma}$ could be reduced to

$$B_\alpha^\beta|_{SU(3)}^{As} = \epsilon_{\alpha\gamma\eta} T^{[\gamma\eta]\beta},$$

and for the proton $p = B_3^1$ we would have

$$\sqrt{2}|p\rangle_{SU(3)}^{As} = \sqrt{2} B_3^1|_{SU(3)}^{As} = -|udu\rangle + |duu\rangle. \qquad (2.11)$$

Instead the baryon octet based on partly symmetric 8-dimensional tensor $T^{\{\alpha\beta\}\gamma}$ could be written in terms of quark wave functions as

$$\sqrt{6} B_\beta^\alpha|_{SU(3)}^{Sy} = \epsilon_{\beta\delta\eta}\{q^\alpha, q^\delta\} q^\eta.$$

For a proton B_3^1 we have

$$\sqrt{6}|p\rangle_{SU(3)}^{Sy} = \sqrt{6} B_3^1|_{SU(3)}^{Sy} = 2|uud\rangle - |udu\rangle - |duu\rangle. \qquad (2.12)$$

In order to construct fully symmetric spin-unitary spin wave function of the octet baryons in terms of quarks of definite flavour and definite spin projection we should cure only to obtain the overall functions being symmetric under permutations of quarks of all the flavours and of all the spin projections inside the given baryon. (As to the overall asymmetry of the fermion wave function we let colour degree of freedom to assure it.) Multiplying spin wave functions of Eqs.(1.19,1.21) and unitary spin wave functions of Eqs.(2.11,2.12) one gets:

$$\sqrt{18} B_\beta^\alpha|_\uparrow = \sqrt{18}(B_\beta^\alpha|_{SU(3)}^{As} \cdot t_A^j + B_\beta^\alpha|_{SU(3)}^{Sy} \cdot T_S^j). \qquad (2.13)$$

Taking the proton B_3^1 as an example one has

$$\sqrt{18}|p\rangle_\uparrow = |-udu + uud\rangle \cdot |-\uparrow\downarrow\uparrow + \uparrow\uparrow\downarrow\rangle \qquad (2.14)$$

$$|2 \cdot uud - udu - duu\rangle \cdot |2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow\rangle =$$

$$= |2u_\uparrow u_\uparrow d_\downarrow - u_\uparrow d_\uparrow u_\downarrow - d_\uparrow u_\uparrow u_\downarrow$$

$$+2u_\uparrow d_\downarrow u_\uparrow - u_\uparrow u_\downarrow d_\uparrow - d_\uparrow u_\downarrow u_\uparrow + 2d_\downarrow u_\uparrow u_\uparrow - u_\downarrow u_\uparrow d_\uparrow - u_\downarrow d_\uparrow u_\uparrow\rangle.$$

We have used here that

$$|uud\rangle \cdot |\downarrow\uparrow\uparrow\rangle = |u_\downarrow u_\uparrow d_\uparrow\rangle$$

and so on.

Important note. One could safely use instead of the Eq.(2.14) the shorter version but where one already cannot change the order of spinors at all!

$$\sqrt{6}|p\rangle = \sqrt{6}|B_3^1\rangle_\uparrow = |2u_\uparrow u_\uparrow d_\downarrow - u_\uparrow d_\uparrow u_\downarrow - d_\uparrow u_\uparrow u_\downarrow\rangle. \tag{2.15}$$

The wave function of the isosinglet Λ has another structure as one can see oneself upon calcolating

$$2|\Lambda\rangle_\uparrow = -\sqrt{6}|B_3^3\rangle_\uparrow = |d_\uparrow s_\uparrow u_\downarrow + s_\uparrow d_\uparrow u_\downarrow - u_\uparrow s_\uparrow d_\downarrow - s_\uparrow u_\uparrow d_\downarrow\rangle. \tag{2.16}$$

Instead decuplet of baryon resonances $T^{\{\alpha\beta\gamma\}}$ with $J^P = \frac{3}{2}^+$ would be written in the form of a so called weight diagram (it gives a convenient grafic image of the $SU(3)$ IR's on the 2-parameter plane which is characterized by basic elements, in our case the 3rd projection of isospin I_3 as an absciss and hypercharge Y as a ordinate) which for decuplet has the form of a triangle:

$$\begin{array}{c} \Delta^- \quad \Delta^0 \quad \Delta^+ \quad \Delta^{++} \\ \Sigma^{*-} \quad \Sigma^{*0} \quad \Sigma^{*+} \\ \Xi^{*-} \quad \Xi^{*0} \\ \Omega^- \end{array}$$

and has the following quark content:

$$\begin{array}{c} ddd \quad udd \quad uud \quad uuu \\ sdd \quad sud \quad suu \\ ssd \quad ssu \\ sss \end{array}$$

In the $SU(3)$ group all the weight diagram are either hexagones or triangles and often are convevient in applications.

For the octet the weight diagram is a hexagone with the 2 elements in center:

$$\begin{array}{ccc} & n \quad p & \\ \Sigma^- & (\Sigma^0, \Lambda) & \Sigma^+ \\ & \Xi^- \quad \Xi^0 & \end{array}$$

or in terms of quark content:

$$udd \quad uud$$

$$sdd \quad sud \quad suu$$

$$ssd \quad ssu$$

The discouvery of 'charm' put a problem of searching of charm baryons. And indeed they were found! Now we already know $\Lambda_c^+(2285, 1 \pm 0, 6 M\text{э}B)$, $\Lambda_c^+(2625, 6{\pm}0, 8 M\text{э}B)$, $\Sigma_c^{++,+,0}(2455)$, $\Xi_c^{+,0}(2465)$. Let us try to classify them along the IR's of groups $SU(4)$ and $SU(3)$. Now we make a product of three 4-spinors $q^\alpha, \alpha = 1, 2, 3, 4$. Tensor structure is the same as for $SU(3)$. But dimensions of the IR's are certainly others: $4 \times 4 \times 4 = 20_4 + 20'_4 + 20'_4 + \bar{4}$. A symmetrical tensor of the 3rd rank of the dimension $N_n^{SSS} = (n^3 + 3n^2 + 2n)/6$ in $SU(4)$ has the dimension 20 and is denoted usually as 20_4. Reduction of the IR 20_4 in IR's of the group $SU(3)$ has the form $20_4 = 10_3 + 6_3 + 3_3 + 1_3$.

There is an easy way to obtain the reduction in terms of the corresponding dimensions. Really, as it is almost obvious, 4- spinor of $SU(4)$ *reduces to* $SU(3)$ *IR's as* $4_4 = 3_3 + 1_3$. *So the product Eqs.(1.29,1.30) for* $n = 4$ $4_4 \times 4_4 = 10_4 + 6_4$ *would reduce as*

$$(3_3 + 1_3) \times (3_3 + 1_3) = 3_3 \times 3_3 + 3_3 \times 1_3 + 1_3 \times 3_3 + 1_3 \times 1_3 =$$

$$6_3 + \bar{3}_3 + 3_3 + 3_3 + 1_1.$$

As antisymmetric tensor of the 2nd rank $T^{[\alpha\beta]}$ *of the dimension* $N_n^{AA} = n(n-1)/2$ *equal to 6 at n=4 and denoted by us as* 6_4 *should be equal to its conjugate* $T_{[\alpha\beta]}$ *due to the absolutely anisymmetric tensor* $\epsilon_{\alpha\beta\gamma\delta}$ *and so it has the following* $SU(3)$ *content:* $6_4 = 3_3 + \bar{3}_3$. *The remaining symmetric tensor of the 2nd rank* $T^{\{\alpha\beta\}}$ *of the dimension* $N_n^{SS} = n(n+1)/2$ *equal to 10 at n=4 and denoted by us as* 10_4 *would have then* $SU(3)$ *content* $10_4 = 6_3 + 3_3 + 1_3$. *The next step would be to construct reduction to* $SU(3)$ *for products* $6_4 \times 4_4 =$

$20'_4 + \bar{4}_4$ *(see Eq.(1.18) at* $n = 4$*) and* $10_4 \times 4_4 = 20'_4 + 20_4$ *(Eq.(1.20) at* $n = 4$*). Their sum would give us the answer for* $4 \times 4 \times 4$.

$$6_4 \times 4_4 = (3_3 + \bar{3}_3) \times (3_3 + 1_3) = \bar{4}_4 + 20'_4 =$$

$$= 3_3 \times 3_3 + \bar{3}_3 \times 3_3 + 3_3 + \bar{3}_3 =$$

$$= (\bar{3}_3 + 1_3) + (8_3 + 6_3 + \bar{3}_3 + 3_3.)$$

As $20'_4$ *is now known it easy to obtain* 20_4*:*

$$20_4 = 10_4 \times 4_4 - 20'_4 = 10_3 + 6_3 + 3_3 + 1_3.$$

Tensor calculus with $q^\alpha = \delta^\alpha_a q^a + \delta^\alpha_4 q^4$ *would give the same results.*

So 20_4 contains as a part 10-plet of baryon resonances $3/2^+$. As to the charm baryons $3/2^+$ there are two candidate: $\Sigma_c(2520)$ and $\Xi_c(2645)$ (J^P has not been measured; $3/2^+$ is the quark model prediction) Note by the way that J^P of the Ω^- which manifested triumph of $SU(3)$ has not been measured since 1964 that is for more than 40 years! Nevertheless everybody takes for granted that its spin-parity is $3/2^+$.

Instead baryons $1/2^+$ enter $20'$-plet described by traceless tensor of the 3rd rank of mixed symmetry $B^\alpha_{[\gamma\beta]}$ antisymmetric in two indices in square brackets:

$$\sqrt{6} B^\alpha_{[\gamma\beta]} = \epsilon_{\gamma\beta\delta\eta}\{q^\alpha, q^\delta\} q^\eta$$

$\alpha, \beta, \gamma, \delta, \eta = 1, 2, 3, 4$. This $20'$-plet as we have shown is reduced to the sum of the $SU(3)$ IR's as $20'_4 = 8_3 + 6_3 + 3_3 + \bar{3}_3$. It is convenient to choose reduction along the multiplets with the definite value of charm. Into 8-plet with $C = 0$ the usual baryon octet of the quarks u,d,s (2.1) is naturally placed. The triplet should contain not yet discouvered in a definite way doubly-charmed baryons:

$$\begin{array}{cc} \Xi^+_{cc} & \Xi^{++}_{cc} \\ \multicolumn{2}{c}{\Omega^+_{cc}} \end{array}$$

with the quark content

$$\begin{array}{cc} ccd & ccu \\ \multicolumn{2}{c}{ccs,} \end{array}$$

and, for example, the wave function of Ξ^+_{cc} in terms of quarks reads

$$\sqrt{6}|\Xi^+_{cc}\rangle_\uparrow = |2c_\uparrow c_\uparrow u_\downarrow - c_\uparrow u \uparrow c_\downarrow - u_\uparrow c_\uparrow c_\downarrow\rangle. \qquad (2.17)$$

Antitriplet contains already discouvered baryons with $C = 1$

$$\Lambda_c^+$$

$$\Xi_c^0 \quad \Xi_c^+$$

with the quark content

$$udc$$

$$dsc \quad usc,$$

and, for example, the wave function of Ξ_c^+ in terms of quarks reads

$$2|\Xi_c^+\rangle_\uparrow = |s_\uparrow c_\uparrow u_\downarrow + c_\uparrow s_\uparrow u_\downarrow - u_\uparrow c_\uparrow s_\downarrow - c_\uparrow u_\uparrow s_\downarrow\rangle. \tag{2.18}$$

Sextet contains discouvered baryons with $C = 1$

$$\Sigma_c^0 \quad \Sigma_c^+ \quad \Sigma_c^{++}$$

$$\Xi_c^{\prime 0} \quad \Xi_c^{\prime +}$$

$$\Omega_c^0$$

(Note that their quantum numbers are not yet measured!) with the quark content

$$ddc \quad udc \quad uuc$$

$$dsc \quad usc$$

$$ssc$$

and, for example, the wave function of $\Xi_c^{\prime +}$ in terms of quarks reads

$$\sqrt{12}|\Xi_c^{\prime +}\rangle_\uparrow = |2u_\uparrow s_\uparrow c_\downarrow + 2c_\uparrow u_\uparrow d_\downarrow \tag{2.19}$$

$$-u_\uparrow c_\uparrow s_\downarrow - c_\uparrow u_\uparrow s_\downarrow - s_\uparrow c_\uparrow u_\downarrow - c_\uparrow s_\uparrow u_\downarrow\rangle.$$

Note also that in the $SU(4)$ group absolutely antisymmetric tensor of the 4th rank $\epsilon_{\gamma\beta\delta\eta}, \gamma, \beta, \delta, \eta$, transforms as singlet IR and because of that antisymmetric tensor of the 3th rank in $SU(4)$ $T^{[\alpha\beta\gamma]}, \alpha, \beta, \gamma = 1, 2, 3, 4$, transforms not as a singlet IR as in $SU(3)$ (what is proved in $SU(3)$ by reduction with the tensor $\epsilon_{\beta\delta\eta}, \beta, \delta, \eta = 1, 2, 3$) but instead along the conjugated spinor representation $\bar{4}$ (which also can be proved by the reduction of it with the tensor $\epsilon_{\gamma\beta\delta\eta}, \gamma, \beta, \delta, \eta = 1, 2, 3, 4$).

We return here to the problem of reduction of the IR of some group into the IR's of minor group or, in particular, to IR's of a production of two minor groups. Well-known example is given by the group $SU(6) \supset SU(3) \times SU(2)_S$ *which in nonrelativistic case has unified unitary model group* $SU(3)$ *and spin group* $SU(2)_S$. *In the framework of* $SU(6)$ *quarks belong to the spinor of dimension* 6_6 *which in the space of* $SU(3) \times SU(2)_S$ *could be written as* $6_6 = (3, 2)$ *where the first symbol in brackets means 3-spinor of* $SU(3)$ *while the 2nd symbol just states for 2-dimensional spinor of* $SU(2)$. *Let us try now to form a product of 6-spinor and corresponding 6-antispinor and reduce it to IR's of the product* $SU(3) \times SU(2)_S$*:*

$$\bar{6}_6 \times 6_6 = 35_6 + 1_6 = (\bar{3}, 2) \times (3, 2) = (\bar{3} \times 3, 2 \times 2) =$$

$$= (8 + 1, 3 + 1) = [(8, 1) + (8, 3) + (1, 3)] + (1, 1),$$

that is, in 35_6 *there are exactly eight mesons of spin zero and 9=8+1 vector mesons, while there is also zero spin meson as a* $SU(6)$ *singlet. This result suits nicely experimental observations for light (of quarks* u, d, s*) mesons. In order to proceed further we form product of two 6-spinors first according to our formulae, just dividing the product into symmetric and antisymmetric IR's of the rank 2:*

$$6_6 \times 6_6 = 21_6 + 15_6 = (3, 2) \times (3, 2) = (3 \times 3, 2 \times 2) =$$

$$= (6_3 + \bar{3}_3, 3 + 1) =$$

$$\{(6_3, 3) + (3_3, 1)\}_{21} + [(6_3, 1) + (\bar{3}_3, 3)]_{15},$$

dimensions of symmetric tensors of the 2nd rank being $n(n + 1)/2$ *while of those antisymmetric* $n(n-1)/2$. *Now we go to product* $15_6 \times 6_6$ *which should results as already we have seen in the sum of two IR's of the 3rd rank , one of them being antisymmetric of the 3rd rank* ($N^{AAA} = n(n^2 - 3n + 2)/6$) *and the other being of mixed symmetry* ($N_{mix} = n(n^2 - 1)/3$)*:*

$$15_6 \times 6_6 = 20_6 + 70_6 = [(6_3, 1) + (\bar{3}_3, 3)] \times (3, 2) =$$

$$= (6 \times 3, 2) + (\bar{3} \times 3, 3 \times 2) =$$

$$= [(8, 2) + (1, 4)]_{20} + [(8, 4) + (10, 2) + (8, 2) + (1, 2)]_{70}.$$

Instead the product $21_6 \times 6_6$ *should result as already we have seen into the sum of two IR's of the 3rd rank , one of them being symmetric of the 3rd* *rank* ($N^{SSS} = n(n^2+3n+2)/6$) *and the other being again of mixed symmetry* ($N_{mix} = n(n^2-1)/3$) *and we used previous result to extract the reduction of the* 56_6*-plet:*

$$21_6 \times 6_6 = 56_6 + 70_6 = \{(6_3, 3) + (\bar{3}_3, 1)\} \times (3, 2) =$$

$$= (6_3 \times 3_3, 3 \times 2) + (\bar{3}_3 \times 3, 1 \times 2) =$$
$$= \{((8,2) + (10,4))\}_{56} + [(8,2) + (1,4)]_{20} +$$
$$+[(8,4) + (10,2) + (8,2) + (1,2)]_{70}.$$

We now see eminent result of $SU(6)$ *that is that in one IR* 56_6 *there are octet of baryons of spin 1/2 and decuplet of baryonic resonances of spin 3/2! Note that quark model with all the masses, magnetons etc equal just reproduces* $SU(6)$ *model as it should be. In some sense 3-quark model gives the possibility of calculations alternative to tensor calculus of* $SU(6)$ *group.*

Some words also on reduction of IR of some group to IR's of the sum of minor groups. We have seen an example of reduction of the IR of $SU(4)$ *into those of* $SU(3)$*. For future purposes let us consider some examples of the reduction of the IR's of* $SU(5)$ *into those of the direct sum* $SU(3)$+$SU(2)$*. Here we just write 5-spinor of* $SU(5)$ *as a direct sum:* $5_5 = (3_3, 1) + (1_3, 2)$*. Forming the product of two 5-spinors we get:*

$$5_5 \times 5_5 = 15_5 + 10_5 = (3_3, 1) + (1_3, 2) \times (3_3, 1) + (1_3, 2) =$$

$$= (3_3 \times 3_3, 1) + (3_3, 2) + (3_3, 2) + (1_3, 2 \times 2) =$$
$$\{(6,1) + (3_3, 2) + (1,3)\}_{15} + [(3_3, 2) + (\bar{3}_3, 1) + (1,1)]_{10},$$

that is, important for $SU(5)$ *group IR's of dimensions 5 and 10 have the following reduction to the sum* $SU(3)$+$SU(2)$*:*

$$5_5 = (3_3, 1) + (1_3, 2),$$

$$10_5 = (3_3, 2) + (\bar{3}_3, 1) + (1, 1).$$

In this case it is rather easy an exercise to proceed also with tensor calculus.