



**NeuAstroSch\_2010**

**Научный семинар  
по программе Школы молодых ученых  
“Физика нейтрино  
и астрофизика”**

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А.И.Студеникин (МГУ) – председатель

**29 сентября 2010 г.  
Физический факультет МГУ  
ЮФА (начало в 17.00)  
Лекцию**

**«Электромагнитные свойства нейтрино: окно в новую физику»**  
прочтёт профессор А.И.Студеникин  
(кафедра теоретической физики)

# Международная школа

## молодых ученых

### “Физика нейтрино и астрофизика”

[www.icas.ru](http://www.icas.ru)

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А.М.Черепашук (ГАИШ МГУ)

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[www.phys.msu.ru](http://www.phys.msu.ru)
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«Исследование свойств массивных нейтрино с использованием  
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- 3) В.А.Рубаков (ИЯИ РАН),  
«Масса нейтрино и барионная асимметрия Вселенной»
- 4) В.С. Березинский (Лаборатория Гран Сассо, ИНФН),  
«Космологические нейтрино: от обычной к новой физике»
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«Оптические исследования рентгеновских двойных систем»

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**8) В.Н. Лукаш и соавторы (АКЦ ФИАН),**  
**«Космологические ограничения на массу нейтрино»**

# Электромагнитные свойства нейтрино: окно в новую физику

Международная школа для  
молодёжи по физике  
нейтрино и астрофизике,  
29 сентября 2010

Александр Иванович  
Студеникин  
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<b>2001</b>	<b>Андрей Михайлович Егоров</b>	
<b>2006</b>	<b>Артем Юрьевич Иванов</b>	
<b>2003</b>	<b>Владимир Леонидович Кауц</b>	<b>ФИАН</b>
<b>2004</b>	<b>Константин Алексеевич Кузаков</b>	
<b>1995</b>	<b>Геннадий Геннадиевич Лихачев</b>	
<b>2004</b>	<b>Андрей Евгеньевич Лобанов</b>	
<b>2007</b>	<b>Александр Михайлович Савочкин</b>	
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<b>2006</b>	<b>Илья Евгеньевич Трофимов</b>	
<b>2007</b>	<b>Сергей Александрович Шинкевич</b>	
<b>2008</b>	<b>Алексей Викторович Лохов</b>	
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1

Carlo Giunti, Alexander Studenikin :  
“*Neutrino electromagnetic properties*”  
Phys. Atom. Nucl. 73, 2089-2125 (2009),  
*arXiv:0812.3646 v5, Apr 12, 2010*

... within the agreement  
on cooperation between  
Moscow University and  
Instituto Nazionale di  
Fisica Nucleare (INFN)

2

A. Studenikin :  
“*Neutrino magnetic moment: a window to new physics*”  
Nucl. Phys. B (Proc. Supl.) 188, 220 (2009)

3

C. Giunti, A. Studenikin :  
“*Electromagnetic properties of neutrinos*”  
J. Phys.: Conf. Series. 203 (2010) 012100,  
*arXiv:1006.3646 June 8, 2010*

4

C. Giunti, A. Studenikin : “*Theory and phenomenology  
of neutrino electromagnetic properties*”  
Rev. Mod. Phys. (in preparation)

*...Why*

# Electromagnetic properties of

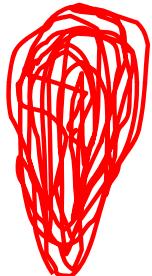
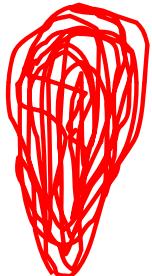
*provide a kind of window / bridge to*

NEW Physics ?

*... up to now, in spite of reasonable efforts,*

- ***NO** any unambiguous experimental confirmation in favour of nonvanishing  $\nu$  em properties ,*
- *available experimental data in the field does not rule out possibility that  $\nu$  have “ZERO” em properties.*
- *... However, in course of recent development of knowledge on  $\nu$  mixing and oscillations,*

Recent studies (exp. & theor.) of  
**flavour conversion of**  
**solar, atmospheric, reactor and accelerator**  
**neutrinos** have conclusively established that



**neutrinos have non-zero mass**  
and they **mix among themselves**  
that provides the first evidence of **new physics**  
beyond the standard model

В. Грибов, Б. Понтеорво (1965)

С. Биленкин, Б. Понтеорво (1976)

## \* Осциляции $\nu$ в вакууме

$$\nu^f = \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \leftrightarrow \nu^P = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

состояния  
взаимодействия

массовые состояния  
 $(m_1, m_2)$

$(\bar{\pi}^+ \rightarrow \mu^+ + \nu_\mu)$   
 $(n \rightarrow p + e^- + \bar{\nu}_e)$

$$\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta$$
$$\nu_\mu = -\nu_1 \sin \theta + \nu_2 \cos \theta$$

угол смешивания  
нейтрино в  
вакууме

$$U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

$$\nu_e = \nu_1 \cos \theta + \nu_2 \sin \theta$$

$$\nu_\mu = -\nu_1 \sin \theta + \nu_2 \cos \theta$$

угол смешивания  
нейтрино в  
вакууме  
матрица  
смешивания

$$U = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix}$$

## Эволюция путька $\nu$ во времени (пространстве)

$$i \frac{d}{dt} \nu^P(t) = H \nu^P(t), \quad H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}, \quad E_i \approx |\vec{p}| + \frac{m_i^2}{2|\vec{p}|}$$

$\checkmark$

$$P_{\nu_e \rightarrow \nu_\mu} (x) = \sin^2 2\theta \sin^2 \frac{\pi x}{L}$$

путь,  
пройденный  
нейтрино  
энергия  
нейтрино

осциляции  
нейтрино

амплитуда  
осциляций

длина осциляций

$$L = \frac{4\pi E}{\Delta m^2}, \quad \Delta m^2 = m_2^2 - m_1^2$$

$$K = \omega_{23} \cdot \omega_{13} \cdot \omega_{12}$$

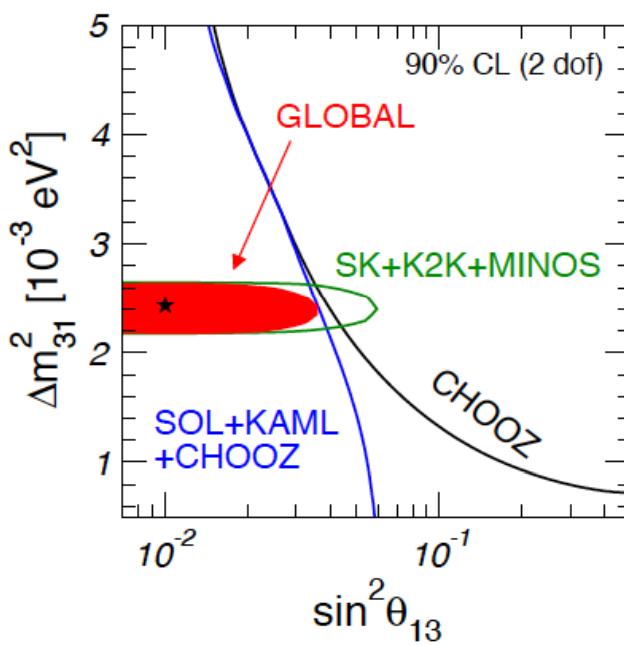
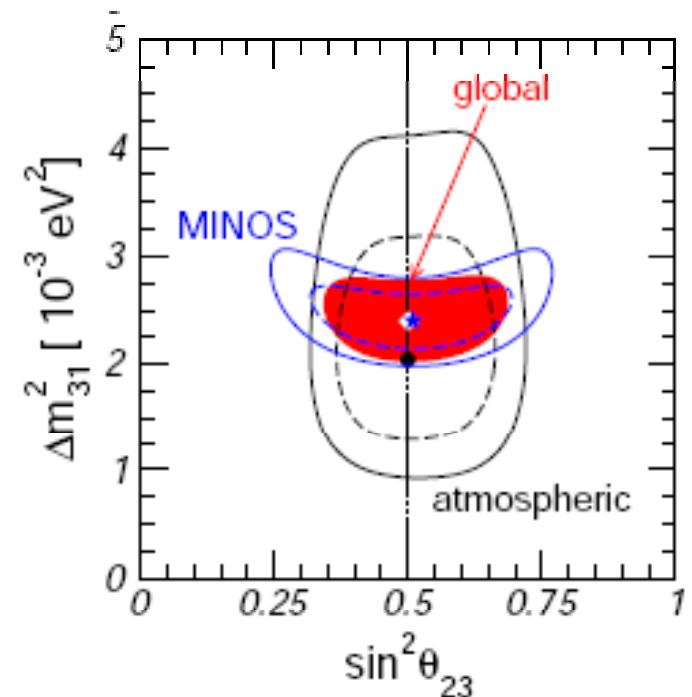
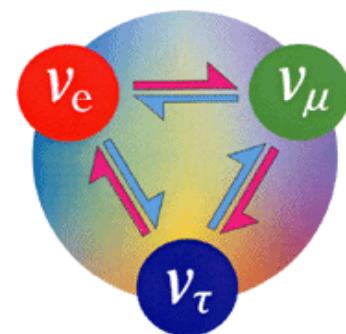
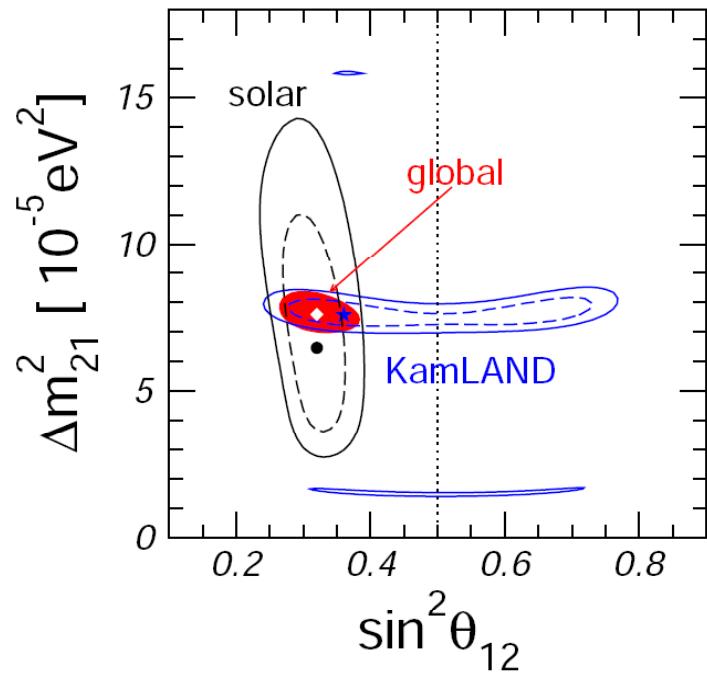
PDG

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & e^{i\phi_{23}} s_{23} \\ 0 & -e^{-i\phi_{23}} s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & e^{i\phi_{13}} s_{13} \\ 0 & 1 & 0 \\ -e^{-i\phi_{13}} s_{13} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & e^{i\phi_{12}} s_{12} & 0 \\ -e^{-i\phi_{12}} s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

23=atm+acc
13=reactor + ..
12=solar+KL

- Even in its simplest unitary form K **differs** from quark mixing matrix, with two extra (Majorana) phases
- In seesaw-schemes K is **not** unitary => extra an& phases => NSI, new propagation effects & LFV among charged leptons
- We assume K real unitary in description of oscillations

Schwetz et al, NJP 10 (2008) 113011



**Homestake, SAGE  
GALLEX/GNO,  
Super-K,  
Borexino  
KamLAND (180 Km)**

**... Super-K**

**K2K (250 Km)  
MINOS  
(735 Km)**

Table I: Best fit values from global data  
 (solar, atmospheric, reactor (KamLand  
 and CHOOZE) and K2K experiments)

parameter	best fit	$2\sigma$	$3\sigma$
$\Delta m_{21}^2$ [ $10^{-5}$ eV $^2$ ]	$7.59^{+0.23}_{-0.18}$	7.22–8.03	7.03–8.27
$ \Delta m_{31}^2 $ [ $10^{-3}$ eV $^2$ ]	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2 \theta_{12}$	$0.318^{+0.019}_{-0.016}$	0.29–0.36	0.27–0.38
$\sin^2 \theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2 \theta_{13}$	$0.013^{+0.013}_{-0.009}$	$\leq 0.039$	$\leq 0.053$

# Neutrino mass



$m_\nu \neq 0 !$

## Neutrino magnetic moment

$\mu_\nu \neq 0$

\* { Lee Shrock } 1977  
Fujikawa } 1980

... Massive neutrino electromagnetic properties ...

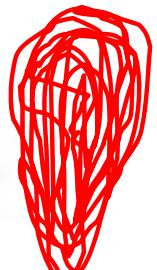
$\nu$   
○

## Theory ( Standard Model with $\nu_R$ )

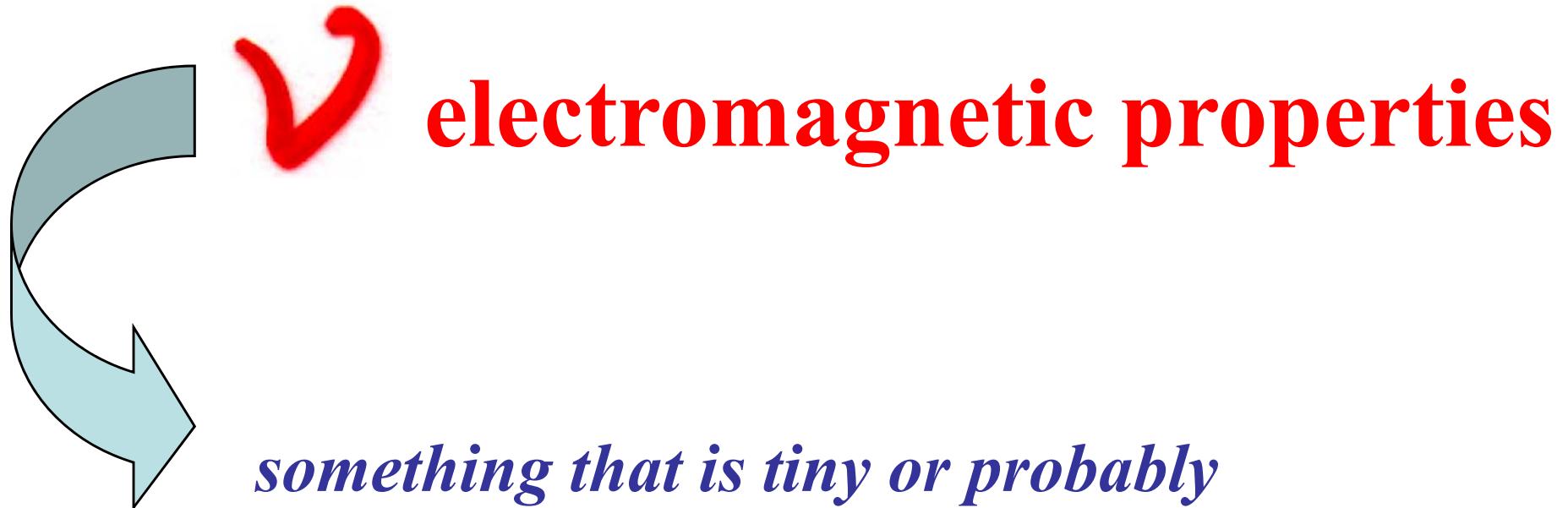
$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_e \sim 3 \cdot 10^{-19} \mu_B \left( \frac{m_{\nu_e}}{1 \text{ eV}} \right), \quad \mu_B = \frac{e}{2m_e}$$

Lee Fujikawa  
Shrock, 1977; Shrock, 1980

In the Standard Model :  $m_\nu = 0$ ,  
there is no  $\nu_R \Rightarrow$   
 $\nu$  magnetic moment  $\mu_\nu = 0$ .  
Thus,  $\mu_\nu \neq 0 \rightarrow$  beyond the SM.



*... puzzling*



*something that is tiny or probably  
even does not exist at all...*

$\nu$

## exhibits unexpected properties (puzzles)

W. Pauli, 1930

- neutron
  - neutral
  - and probably
  - massless particle
  - $\nu$  very important player (astrophysics, cosmology etc. . .)
- now we know that it is **neutrino** E.Fermi,  
1933
- now we know that  $q_\nu \neq 0$  in plasma and beyond SM (?)
- $m_\nu \neq 0$  ? !
- now we know that  $m_\nu \neq 0$

Pauli himself wrote to Baade:

*“Today I did something a physicist should never do.  
I predicted something which will never be observed  
experimentally...”.*

*... we very much hope that*

**V** electromagnetic properties

*will not follow the presentiment of Pauli*

# Outline (short list)

- $\nu$  electromagnetic properties - theory
- $\nu$  magnetic moment - experiment
- constraints on  $\nu$  electromagnetic properties
- $\nu$  electromagnetic interactions  
( $\nu$ - $\gamma$  processes )

# ● Outline ●

## 0. Introduction

### 1. $\nu$ magnetic moment in experiments

### 2. New experimental result on $\mu_\nu$

### 3. $\nu$ electromagnetic properties - theory

#### 3.1 $\nu$ vertex function

#### 3.2 $\mu_\nu$ (arbitrary masses)

#### 3.3 relationship between $m_\nu$ and $\mu_\nu$

#### 3.4 $\nu$ vertex function in case of flavour mixing

#### 3.5 $\nu$ dipole moments in case of mixing

#### 3.6 $\mu_\nu$ in left-right symmetry models

#### 3.7 astrophysical bounds on $\mu_\nu$

#### 3.8 $\nu$ millicharge (Red Giants cooling etc)

#### 3.9 $\nu$ charge radius and anapole moment

#### 3.10 $\nu$ electromagnetic properties in matter and e.m.f.

## 4. Effects of $\nu$ electromagnetic properties

#### 3.11 $\nu$ radiative decay, Ch radiation and Spin Light of $\nu$ in matter

#### 3.12 $\nu$ radiative $2\nu\bar{\nu}$ -decay

#### 3.13 $\nu$ spin-flavour oscillations

## 5. Direct-Indirect influence of e.m.f. on $\nu$

## 6. Conclusion

Giunti, Studenikin :

*“Neutrino electromagnetic properties”*

arXiv:0812.3646,

Phys.Atom.Nucl. 73 (2009)

Studenikin :

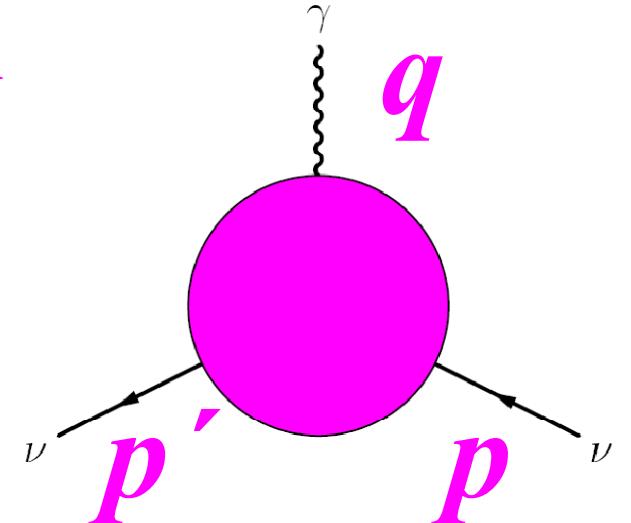
*“Neutrino magnetic moment: a window to new physics”*

arXiv:0812.4716,

Nucl.Phys.B (Proc.Supl.) 188,  
220 (2009)

# $\mathcal{V}$ electromagnetic vertex function

$$\langle \psi(p') | J_\mu^{EM} | \psi(p) \rangle = \bar{u}(p') \Lambda_\mu(q, l) u(p)$$



*Matrix element of electromagnetic current  
is a Lorentz vector*

## Lorentz covariance (1)

$\Lambda_\mu(q, l)$  should be constructed using

**matrices**  $\hat{1}, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu},$

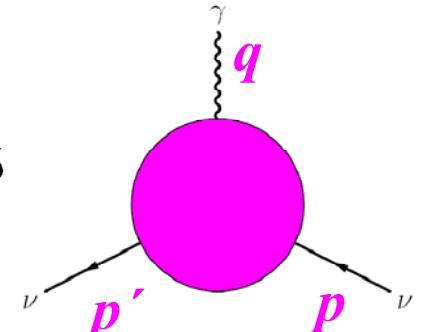
**tensors**  $g_{\mu\nu}, \epsilon_{\mu\nu\sigma\gamma}$

**vectors**  $q_\mu$  and  $l_\mu$

$$q_\mu = p'_\mu - p_\mu, \quad l_\mu = p'_\mu + p_\mu$$

**Vertex function**  $\Lambda_\mu(q, l)$   **there are three sets of operators:**

- $\hat{1}q_\mu, \hat{1}l_\mu, \gamma_5 q_\mu, \gamma_5 l_\mu$
- $\not{q}q_\mu, \not{l}q_\mu, \gamma_5 q_\mu, \gamma_5 \not{q}q_\mu, \gamma_5 \not{l}q_\mu, \sigma_{\alpha\beta} q^\alpha l^\beta q_\mu, \{q_\mu \leftrightarrow l_\mu\}$
- $\gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu} q^\nu, \sigma_{\mu\nu} l^\nu.$
- $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} q^\nu, \epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} l^\nu, \epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} q_\beta q^\sigma l^\gamma,$   
 $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} l_\beta q^\sigma l^\gamma, \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \hat{1}, \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \gamma_5$



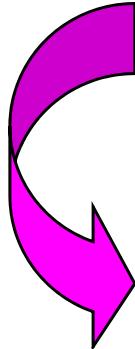
↙ **vertex function (using Gordon-like identities)**

$$\begin{aligned} \Lambda_\mu(q, l) = & f_1(q^2)q_\mu + f_2(q^2)q_\mu \gamma_5 + f_3(q^2)\gamma_\mu + \\ & f_4(q^2)\gamma_\mu \gamma_5 + f_5(q^2)\sigma_{\mu\nu} q^\nu + f_6(q^2)\epsilon_{\mu\nu\rho\gamma} \sigma^{\rho\gamma} q^\nu, \end{aligned}$$

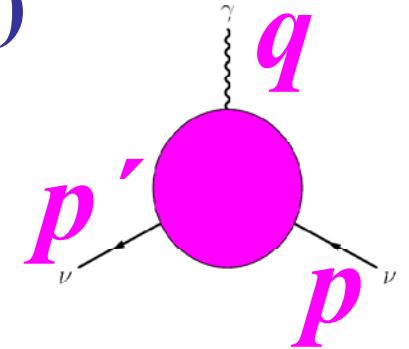
the only dependence on  $q^2$  remains because  $p^2 = p'^2 = m^2, l^2 = 4m^2 - q^2$

# Electromagnetic gauge invariance (2)

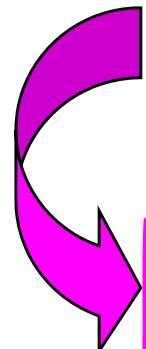
(requirement of current conservation)



$$\partial_\mu j^\mu = 0$$



$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0,$$



$$f_1(q^2) = 0, \quad f_2(q^2)q^2 + 2mf_4(q^2) = 0$$



**v**

vertex function

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu\gamma_5)\gamma_5$$

charge

dipole electric and magnetic

anapole

... consistent with  
Lorentz-covariance (1)

+

electromagnetic gauge invariance (2)

4 Form Factors

# Matrix element of electromagnetic current between neutrino states

$$\langle \nu(p') | J_\mu^{EM} | \nu(p) \rangle = \bar{u}(p') \Lambda_\mu(q) u(p).$$

where vertex function generally contains **4 form factors**

$$\Lambda_\mu(q) = f_Q(q^2) \gamma_\mu + f_M(q^2) i \sigma_{\mu\nu} q^\nu - f_E(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + f_A(q^2) (q^2 \gamma_\mu - q_\mu q^\nu) \gamma_5$$

1. electric   dipole   2. magnetic   3. electric   4. anapole



Hermiticity and discrete symmetries of EM current  $J_\mu^{EM}$  put constraints on form factors

**Dirac**

- 1)  $CP$  invariance + hermiticity  $\implies f_E = 0$ ,
- 2) at zero momentum transfer only electric charge  $f_Q(0)$  and magnetic moment  $f_M(0)$  contribute to  $H_{int} \sim J_\mu^{EM} A^\mu$ ,
- 3) hermiticity itself  $\implies$  three form factors are real:  $Im f_Q = Im f_M = Im f_A = 0$ .



**Majoran**

- 1) from  $CPT$  invariance (regardless  $CP$  or ~~CP~~).

$$f_Q = f_M = f_E = 0$$

↑      ↑

...as early as 1939, W.Pauli...



EM properties a way to distinguish **Dirac** and **Majorana**

In general case matrix element of  $J_\mu^{\text{EM}}$  can be considered between different initial  $\psi_i(p)$  and final  $\psi_j(p')$  states of different masses  $p^2 = m_i^2$ ,  $p'^2 = m_j^2$ :



$$\langle \psi_j(p') | J_\mu^{\text{EM}} | \psi_i(p) \rangle = \bar{u}_j(p') \Lambda_\mu(q) u_i(p)$$

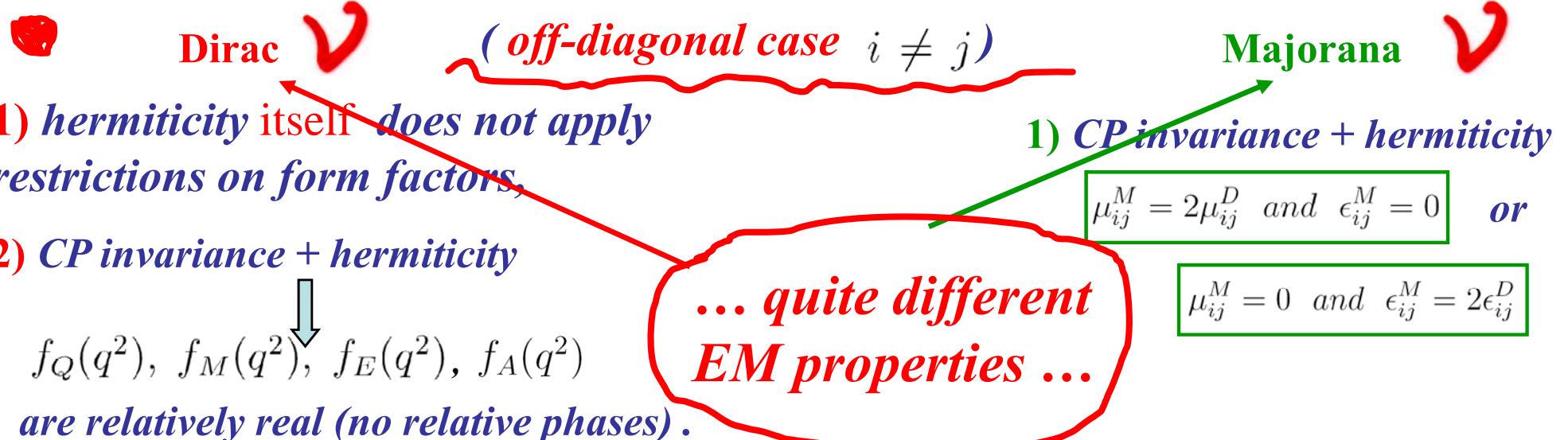
... beyond  
SM...

and

$$\Lambda_\mu(q) = \left( f_Q(q^2)_{ij} + f_A(q^2)_{ij} \gamma_5 \right) (q^2 \gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i \sigma_{\mu\nu} q^\nu + f_E(q^2)_{ij} \sigma_{\mu\nu} q^\nu \gamma_5$$



form factors are matrices in mass eigenstates space.





magnetic moment ?

Dipole **magnetic**

$$f_M(q^2)$$

and **electric**

$$f_E(q^2)$$

are most well studied and theoretically understood  
among form factors

...because even in the limit

$$q^2 \rightarrow 0$$

they may have

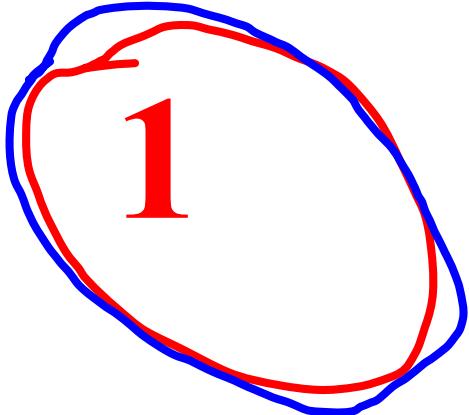
nonvanishing values

$$\mu_\nu = f_M(0)$$

$\nu$  magnetic moment

$$\epsilon_\nu = f_E(0)$$

$\nu$  electric moment ???



# *v magnetic moment in experiments*

Samuel Ting

(wrote on the wall at Department of Theoretical Physics of Moscow State University) :

“*Physics is an experimental science*”

# Studies of $\nu$ - $e$ scattering - most sensitive method of experimental investigation of $\mu_\nu$

Cross-section:

- $$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left( \frac{d\sigma}{dT} \right)_{SM} + \left( \frac{d\sigma}{dT} \right)_{\mu_\nu},$$

where the Standard Model contribution

- $$\left( \frac{d\sigma}{dT} \right)_{SM} = \frac{G_F^2 m_e}{2\pi} \left[ (g_V + g_A)^2 + (g_V - g_A)^2 \left( 1 - \frac{T}{E_\nu} \right)^2 + (g_A^2 - g_V^2) \frac{m_e T}{E_\nu^2} \right],$$

$T$  is the electron recoil energy and

- $$\left( \frac{d\sigma}{dT} \right)_{\mu_\nu} = \frac{\pi \alpha_{em}^2}{m_e^2} \left[ \frac{1 - T/E_\nu}{T} \right] \mu_\nu^2$$

- $$\mu_\nu^2 = \sum_{j=\nu_e, \nu_\mu, \nu_\tau} |\mu_{ij} - \epsilon_{ij}|^2$$

$$g_V = \begin{cases} 2 \sin^2 \theta_W + \frac{1}{2} & \text{for } \nu_e, \\ 2 \sin^2 \theta_W - \frac{1}{2} & \text{for } \nu_\mu, \nu_\tau, \end{cases} \quad g_A = \begin{cases} \frac{1}{2} & \text{for } \nu_e, \\ -\frac{1}{2} & \text{for } \nu_\mu, \nu_\tau \end{cases} \quad \text{for anti-neutrinos}$$

,  $g_A \rightarrow -g_A$

to incorporate **charge radius**:  $g_V \rightarrow g_V + \frac{2}{3} M_W^2 \langle r^2 \rangle \sin^2 \theta_W$

- $$\frac{d\sigma}{dT}(\nu + e \rightarrow \nu + e) = \left( \frac{d\sigma}{dT} \right)_{\text{SM}} + \left( \frac{d\sigma}{dT} \right)_{\mu_\nu}$$



$\nu$ - $\gamma$  coupling ... valid for  $\nu$  scattering on free  $e$

- $$\left( \frac{d\sigma}{dT} \right)_{\mu_\nu} = \frac{\pi \alpha_{em}^2}{m_e^2} \left[ \frac{1 - T/E_\nu}{T} \right] \mu_\nu^2$$

with change of helicity,  
contrary to SM

$T$  is the electron recoil energy:

$$0 \leq T \leq \frac{2E_\nu^2}{2E_\nu + m_e}$$

If neutrino has electric dipole moment,  
or electric or magnetic transition moments,  
these quantities would also contribute to scattering cross section

$$\mu_\nu^2 = \sum_{j=\nu_e, \nu_\mu, \nu_\tau} |\mu_{ij} - \epsilon_{ij}|^2 , \quad i \text{ refers to initial neutrino flavour}$$

Possibility of **distractive interference** between **magnetic** and **electric** transition moments of **Dirac** neutrino  
(**Majorana** neutrino has only magnetic or electric transition moment, but not both if CP is conserved)

# Effective $\nu_e$ magnetic moment measured in $\nu$ - $e$ scattering experiments ?

$$\mu_e^2$$

Two steps:

- 1) consider  $\nu_e$  as superposition of mass eigenstates ( $i=1,2,3$ ) at some distance  $L$ , and then sum up magnetic moment contributions to  $\nu$ - $e$  scattering amplitude (of each of mass components) induced by their magnetic moments

$$A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}$$

*J.Beacham,  
P.Vogel, 1999*

- 2) amplitudes combine incoherently in total cross section

$$\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2$$

*C.Giunti,  
A.Studenikin,  
2009*

**NB!** Summation over  $j=1,2,3$  is outside the square because of incoherence of different final mass states contributions to cross section.

# *Effective* $\nu$ magnetic moment in experiments

(for neutrino produced as  $\nu_l$  with energy  $E_\nu$ ,  
and after traveling a distance  $L$ )

$$\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_i L} \mu_{ji} \right|^2$$

where neutrino mixing matrix

$$\mu_{ij} \equiv |\beta_{ij} - \varepsilon_{ij}|$$

magnetic and electric moments

Observable  $\mu_\nu$  is an effective parameter that depends on neutrino flavour composition at the detector.

H.Wong,  
H.-B.Li, 2005

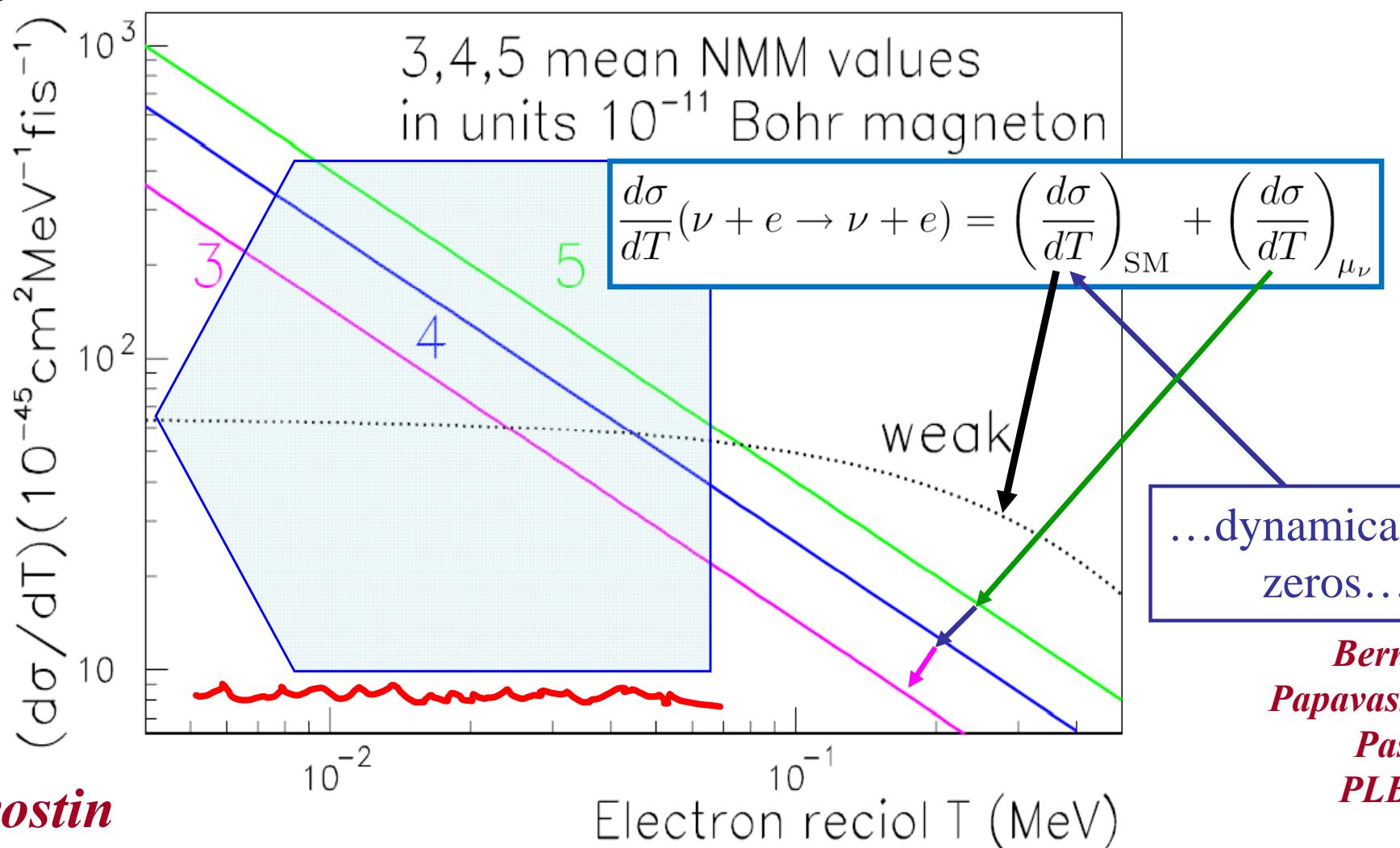
Implications of  $\mu_\nu$  limits from different experiments  
(reactor, solar  ${}^8\text{B}$  and  ${}^7\text{Be}$ ) are different.

Magnetic moment contribution is dominated at low electron recoil energies

and  $\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}$  when  $\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4} \mu_\nu^2$

{ ... the **lower** the smallest measurable electron recoil energy is,

the **smaller** values of  $\mu_\nu^2$  can be probed in scattering experiments ...



from  
*A. Starostin*

*Bernabeu,  
Papavassiliou,  
Passera,  
PLB 2005*

# First and future $\nu$ - $e$ scattering experiments



- 

$$\mu_\nu \leq 2 \div 4 \times 10^{-10} \mu_B$$

Savannah River (1976), *first observation*

Vogel, Engel, 1989

Kurchatov, Krasnoyarsk (1992),  
Rovno (1993) reactors



- 

$$\mu_\nu \leq 1.1 \times 10^{-10} \mu_B$$

SuperKamiokande (2004)

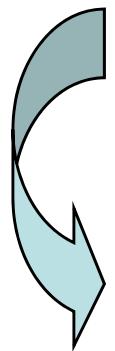
- 

$$\mu_\nu \leq \text{few} \times 10^{-11} \mu_B$$

Beta-beams



McLaughlin, Volpe, 2004



**MUNU** experiment at Bugey reactor (2005)

$$\mu_\nu \leq 9 \times 10^{-11} \mu_B$$



**TEXONO** collaboration at Kuo-Sheng power plant (2006)

$$\mu_\nu \leq 7 \times 10^{-11} \mu_B$$

**GEMMA** (2007)

$$\mu_\nu \leq 5.8 \times 10^{-11} \mu_B$$

**GEMMA I 2005 - 2007**

**BOREXINO** (2008)

$$\mu_\nu \leq 5.4 \times 10^{-11} \mu_B$$

*...was considered as the world best constraint...*

$$\mu_\nu \leq 8.5 \times 10^{-11} \mu_B \quad (\nu_\tau, \nu_\mu)$$



*Montanino,  
Picariello,  
Pulido, PRD 2008  
based on first release of  
BOREXINO data*

2

# GEMMA (2005-2008)

## Germanium Experiment on measurement of Magnetic Moment of Antineutrino

JINR (Dubna) + ITEP (Moscow) at *Kalinin Nuclear Power Plant*



$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$



...till *13 January 2010* and again since *23 August 2010*  
best limit on  $\nu$  magnetic moment

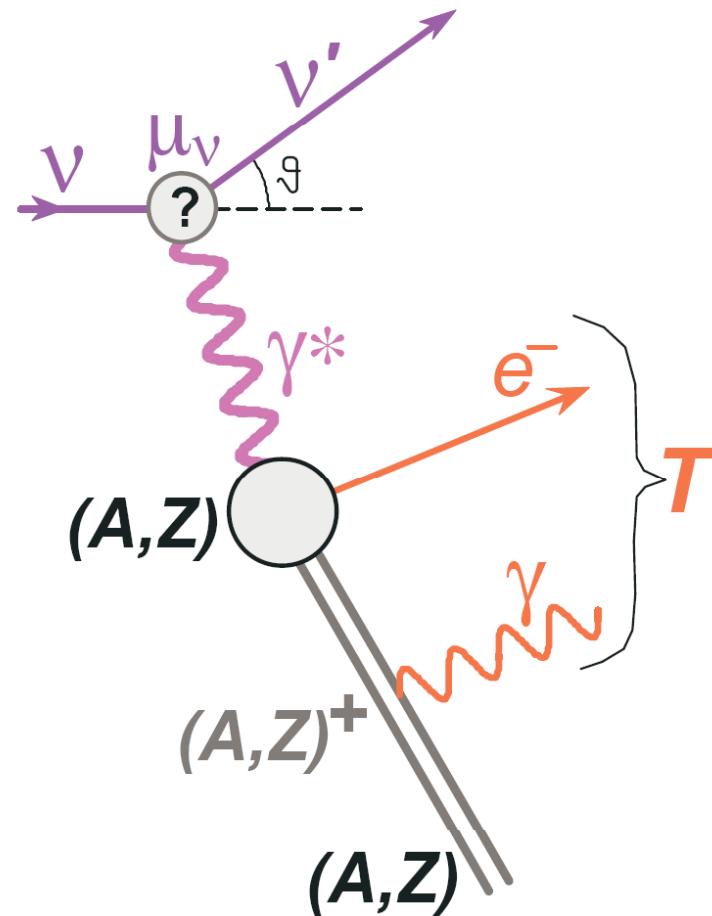
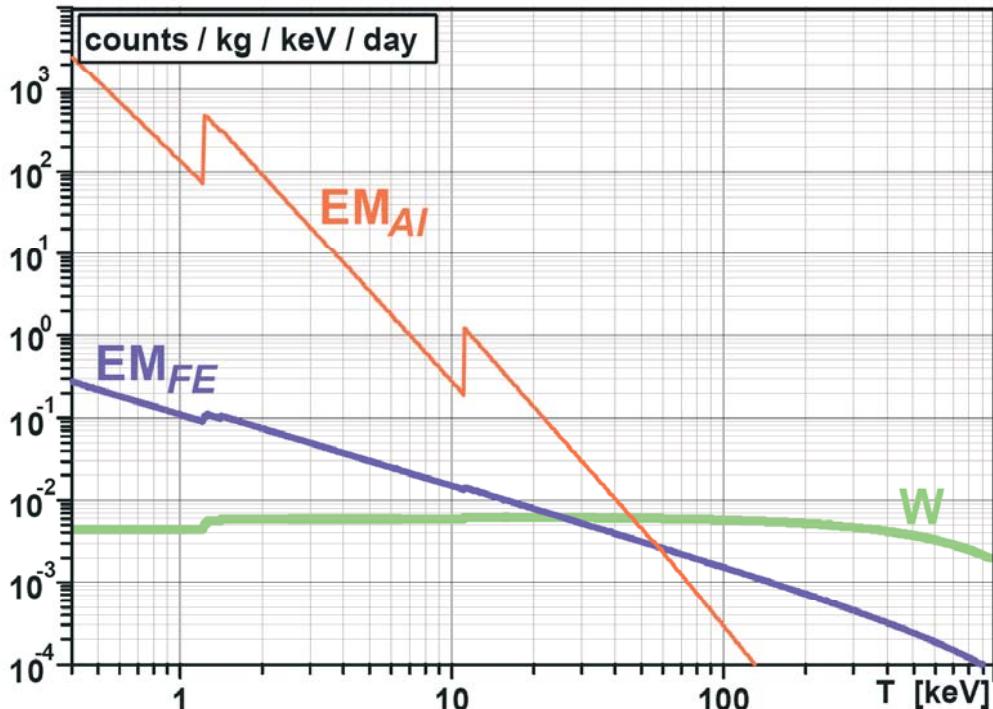
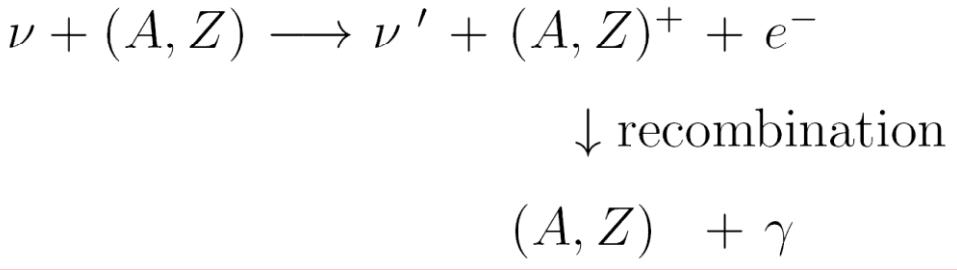
*A.Beda, E.Demidova, A.Starostin, V.Brudanin, V.Egorov, D.Medvedev,  
M.Shirchenko, A.Starostin, Ts.Vylov,  
arXiv:09.06.1926 , June 10, 2009,*

*A.Beda, V.Brudanin, E.Demidova, V.Egorov, G.Gavrilov,  
M.Shirchenko, A.Starostin, Ts.Vylov,  
in: "Particle Physics on the Eve of LHC",  
ed. A.Studenikin, World Scientific (Singapore), p.112, 2009  
[www.icas.ru](http://www.icas.ru) (13th Lomonosov Conference)*

... quite recent claim  
that  $\nu$ - $e$  cross section  
should be increased by  
*Atomic Ionization* effect:

?

*H.Wong et al., arXiv: 1001.2074,  
13 Jan 2010, reported at the  
Neutrino 2010 Conference  
(Athens, June 2010),  
PRL 105 (2010) 061801*



...much better limits on  $\nu$  effective magnetic moment :

$$\mu_\nu < 1.3 \times 10^{-11} \mu_B$$

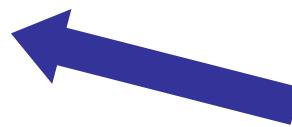


H.Wong et al.,  
arXiv: 1001.2074,

13 Jan 2010,  
PRL 105 (2010) 061801

Neutrino 2010 Conference, Athens

$$\mu_\nu < 5.0 \times 10^{-12} \mu_B$$



(Atomic Ionization effect “accounted for”)

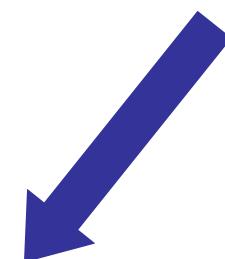
A.Beda et al.  
(GEMMA Coll.),  
arXiv: 1005.2736,  
16 May 2010

... however...

● M.Voloshin, arXiv: 1008.2171, 23 Aug 2010:

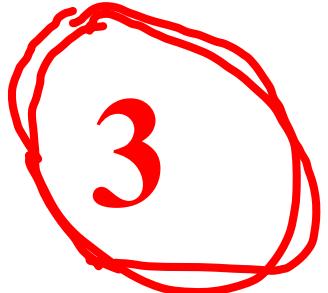
*no important effect of Atomic Ionization on*

$\nu$  cross section once all possible final  
electronic states accounted for



$$\mu_\nu < 3.2 \times 10^{-11} \mu_B$$

( $\nu$ -e scattering on free electrons)



*... a bit of **V** electromagnetic  
properties theory*

3.1

V

vertex function

The most general study of the  
**massive neutrino** vertex function

(including electric and magnetic  
form factors) in arbitrary  $R_S$  gauge

in the context of the  $SM + SU(2)$ -singlet



$\gamma_R$  accounting for masses of particles



in polarization loops

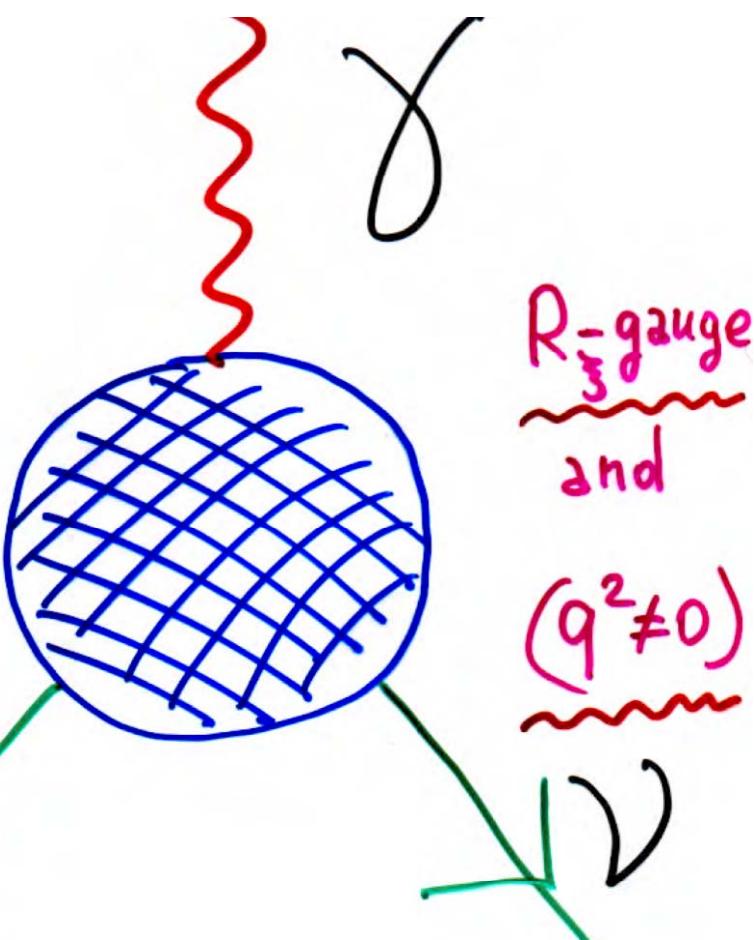


M.Dvornikov, A.Studenikin

\* Phys. Rev. D 63, 073001 2004,

"Electric charge and magnetic moment of massive neutrino";

JETP 126 (2009), N8, 1  
\* "Electromagnetic form factors of a massive neutrino."



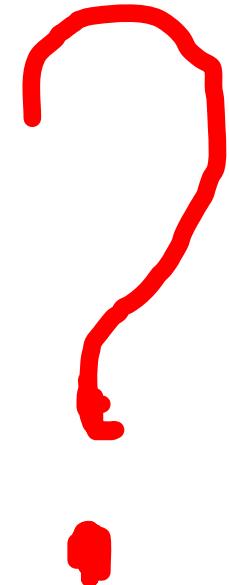
charge

magnetic moment

$$\Delta_\mu(q) = \underbrace{f_Q(q^2)\gamma_\mu}_{\text{electric moment}} + \underbrace{f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)i\sigma_{\mu\nu}q^\nu\gamma_5}_{\text{anapole moment}} - f_A(q^2)(q^2\gamma_\mu - q_\mu\gamma^2)\gamma_5$$

# Magnetic moment dependence

$$\mu_\nu = \mu_\nu(m_\nu)$$



on neutrino mass

$\nu$

# magnetic moment

( for arbitrary neutrino  
mass, heavy neutrino... )

- LEP data



only 3 light  $\nu$ s coupled to  $Z^*$ ,  
for any additional neutrino

$$m_{\nu} \geq 45 \text{ GeV}$$



$m_\nu \ll m_e \ll M_W$

light  $\nu$

$$M_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_e$$

$$\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3} (2 - 7a + 6a^2 - 2a^2 \ln a - a^3) , \quad a = \left(\frac{m_e}{M_W}\right)^2$$

Dvornikov,  
Studenikin,

Phys.Rev.D 69  
(2004) 073001;  
JETP 99 (2004) 254



$m_e \ll m_\nu \ll M_W$

intermediate  $\nu$

Gabral-Rosetti,  
Bernabeu, Vidal,  
Zepeda,  
Eur.Phys.J C 12  
(2000) 633

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{ 1 + \frac{5}{18} b \right\} , \quad b = \left(\frac{m_\nu}{M_W}\right)^2$$



$m_e \ll M_W \ll m_\nu$

$$\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu$$

heavy  $\nu$   
 $\sim 10^{-19} \mu_B \left(\frac{m_{\nu_e}}{1 \text{ eV}}\right)$

...

$\mu_\nu$

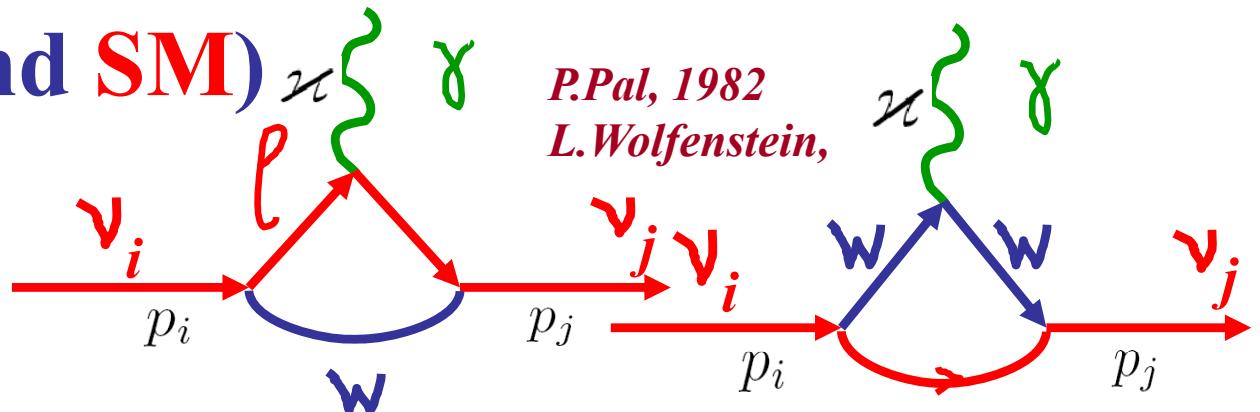
in case of mixing...



3.5

# Neutrino (beyond SM) dipole moments (+ transition moments)

- Dirac neutrino



$$\left. \begin{array}{l} \mu_{ij} \\ \epsilon_{ij} \end{array} \right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2} \left( 1 \pm \frac{m_j}{m_i} \right) \sum_{l=e, \mu, \tau} f(r_l) U_{lj} U_{li}^*$$

$$r_l = \left( \frac{m_l}{m_W} \right)^2$$

$m_e = 0.5 \text{ MeV}$   
 $m_\mu = 105.7 \text{ MeV}$   
 $m_\tau = 1.78 \text{ GeV}$   
 $m_W = 80.2 \text{ GeV}$

- $m_i, m_j \ll m_l, m_W$

$$f(r_l) \approx \frac{3}{2} \left( 1 - \frac{1}{2} r_l \right), \quad r_l \ll 1$$

transition moments vanish  
because unitarity of  $U$   
implies that its rows or columns  
represent orthogonal vectors

- Majorana neutrino  
only for

$$i \neq j$$

$$\mu_{ij}^M = 2\mu_{ij}^D \text{ and } \epsilon_{ij}^M = 0$$

or

$$\mu_{ij}^M = 0 \text{ and } \epsilon_{ij}^M = 2\epsilon_{ij}^D$$

- transition moments are suppressed,  
**Glashow-Iliopoulos-Maiani cancellation,**  
for diagonal moments there is no  
**GIM cancellation**

... depending on relative  
**CP** phase of  $\nu_i$  and  $\nu_j$

# The first nonzero contribution from neutrino transition moments

$$f_{rl} \rightarrow -\frac{3}{2} + \frac{3}{4} \left( \frac{m_l}{m_W} \right)^2 \ll 1$$

GIM cancellation

$$\left. \begin{array}{l} \mu_{ij} \\ \epsilon_{ij} \end{array} \right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2} \left( 1 \pm \frac{m_j}{m_i} \right) \left( \frac{m_\tau}{m_W} \right)^2 \sum_{l=e, \mu, \tau} \left( \frac{m_l}{m_\tau} \right)^2 U_{lj} U_{li}^*$$

$$\mu_B = \frac{e}{2m_e}$$

$$\left. \begin{array}{l} \mu_{ij} \\ \epsilon_{ij} \end{array} \right\} = 4 \times 10^{-23} \mu_B \left( \frac{m_i \pm m_j}{1 \text{ eV}} \right) \sum_{l=e, \mu, \tau} \left( \frac{m_l}{m_\tau} \right)^2 U_{lj} U_{li}^*$$

... neutrino radiative decay is very slow

- Dirac  $\cancel{\nu}$  diagonal ( $i=j$ ) magnetic moment

$$\epsilon_{ii}^D = 0 \quad \text{for } CP\text{-invariant interactions}$$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \left( 1 - \frac{1}{2} \sum_{l=e, \mu, \tau} r_l |U_{li}|^2 \right) \approx 3.2 \times 10^{-19} \left( \frac{m_i}{1 \text{ eV}} \right) \mu_B$$

$$\mu_{ii}^M = \epsilon_{ii}^M = 0$$

Lee, Shrock,  
Fujikawa, 1977

- $\mu_{ii}^D$  - to leading order - independent on  $U_{li}$  and  $m_{l=e, \mu, \tau}$

... possibility to measure fundamental  $\mu_{ii}^D$

$$\mu_e^2 = \sum_{i=1,2,3} |U_{ie}|^2 \mu_{ii}^2$$

$\mu_{ii}^D = 0$  for massless  $\cancel{\nu}$

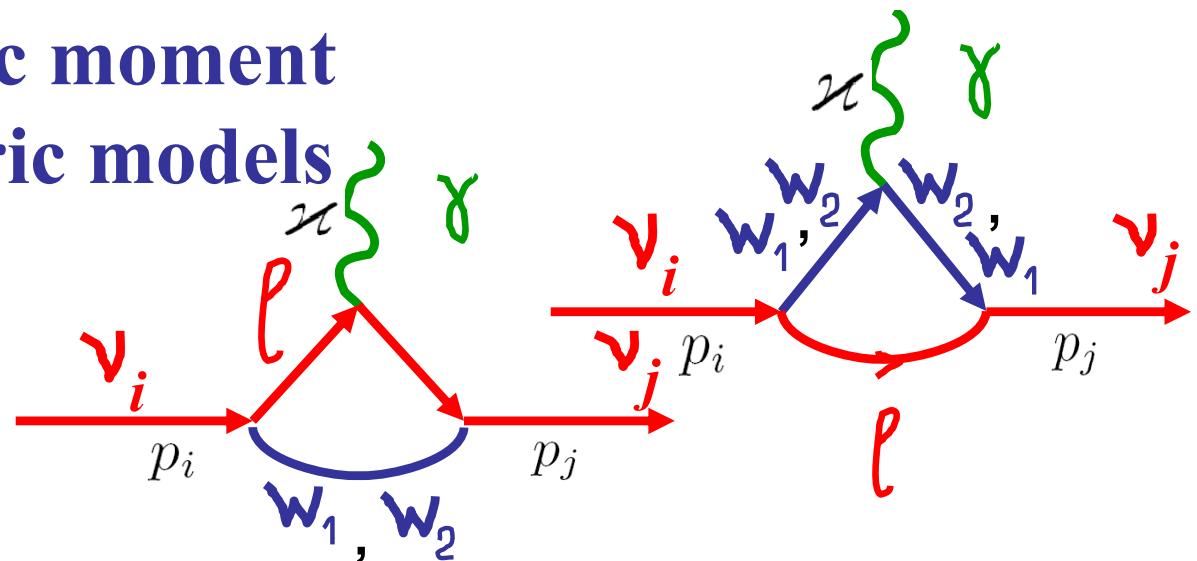
(in the absence of right-handed charged currents)  $\rightarrow$

3.6

## Neutrino magnetic moment in left-right symmetric models

$$SU_L(2) \times SU_R(2) \times U(1)$$

**Gauge bosons**     $W_1 = W_L \cos \xi - W_R \sin \xi$   
**mass states**     $W_2 = W_L \sin \xi + W_R \cos \xi$



with mixing angle  $\xi$  of gauge bosons  $W_{L,R}$  with pure  $(V \pm A)$  couplings

Kim, 1976; Marciano, Sanda, 1977;  
 Beg, Marciano, Ruderman, 1978

$$\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2} \left[ m_l \left( 1 - \frac{m_{W_1}^2}{m_{W_2}^2} \right) \sin 2\xi + \frac{3}{4} m_{\nu_l} \left( 1 + \frac{m_{W_1}^2}{m_{W_2}^2} \right) \right]$$

... charged lepton mass ...

... neutrino mass ...

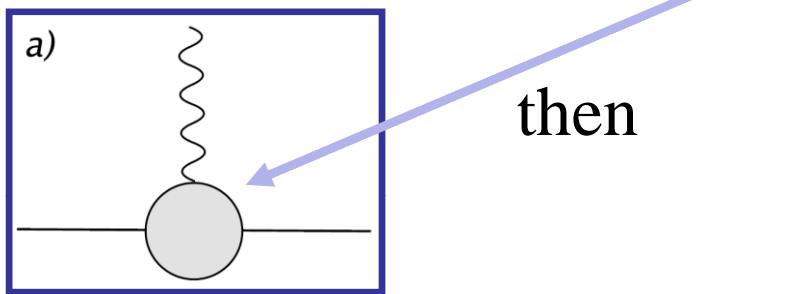
3.3

## Naïve relationship between the size of $m_\nu$ and $\mu_\nu$

*... problem to get large  $\mu_\nu$  and still acceptable  $m_\nu$*

If  $\mu_\nu$  is generated by physics beyond the SM at energy scale  $\Lambda$ ,

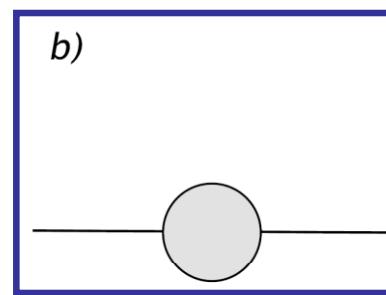
P.Vogel e.a., 2006



then

$$\mu_\nu \sim \frac{eG}{\Lambda},$$

...combination of constants  
and loop factors...



contribution to  $m_\nu$  given by

, then

$$m_\nu \sim G\Lambda$$



Voloshin, 1988;  
Barr, Freire,  
Zee, 1990

$$m_\nu \sim \frac{\Lambda^2}{2m_e \mu_B} \mu_\nu \sim \frac{\mu_\nu}{10^{-18} \mu_B} [\Lambda(\text{TeV})]^2 \text{ eV}$$

from quadratic divergence appearing in renormalization  
of dimension four neutrino mass operator

# Large magnetic moment $\mu_\nu = \mu_\nu(m_\nu, m_B, m_{e^-})$

- In the L-R symmetric models

$$(SU(2)_L \times SU(2)_R \times U(1))$$

Kim, 1976  
Beg, Marciano,  
Ruderman, 1978

- Voloshin, 1988

“On compatibility of small  $m_\nu$ ,  
with large  $\mu_\nu$  of neutrino”,  
Sov.J.Nucl.Phys. 48 (1988) 512

... there may be  $SU(2)_\nu$  symmetry that forbids  $m_\nu$ , but not  $\mu_\nu$

- Bar, Freire, Zee, 1990

*considerable enhancement of  $\mu_\nu$ ,  
to experimentally relevant range*

- supersymmetry

- extra dimensions

- model-independent constraint  $\mu_\nu$

$$\mu_\nu^D \leq 10^{-15} \mu_B$$

$$\mu_\nu^M \leq 10^{-14} \mu_B$$

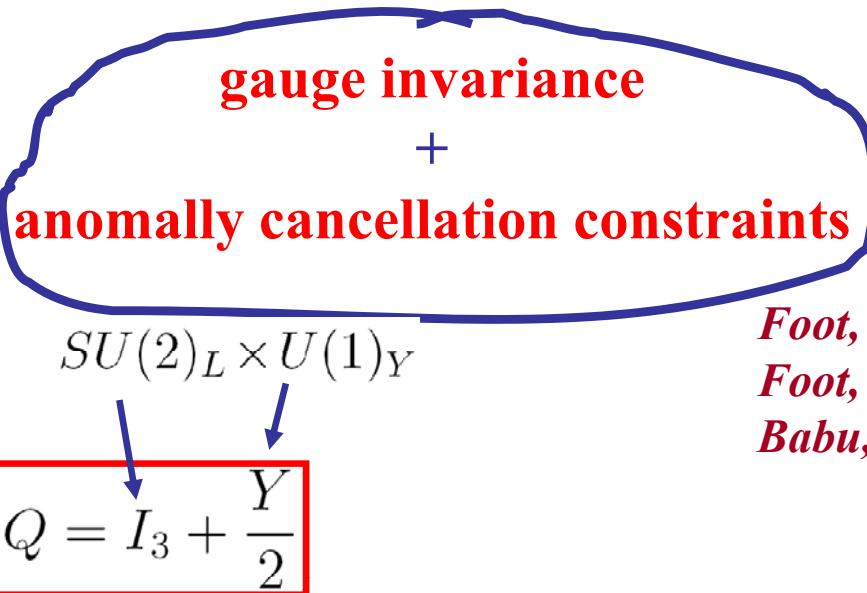
Bell, Cirigliano,  
Ramsey-Musolf,

Vogel,  
Wise,  
2005

for BSM ( $\Lambda \sim 1$  TeV) without fine tuning and  
under the assumption that  $\delta m_\nu \leq 1$  eV

# ... A remark on electric charge of $\nu$ ...

$\nu$  neutrality  $Q=0$  is attributed to



imposed in SM of electroweak interactions

Foot, Joshi, Lew, Volkas, 1990;  
Foot, Lew, Volkas, 1993;  
Babu, Mohapatra, 1989, 1990

...General proof:

- In SM :

- In SM (without  $\nu_R$ ) triangle anomalies cancellation constraints  $\rightarrow$  certain relations among particle hypercharges  $Y$ , that is enough to fix all  $Y$  so that they, and consequently  $Q$ , are quantized
- $Q=0$  is proven also by direct calculation in SM within different gauges and methods

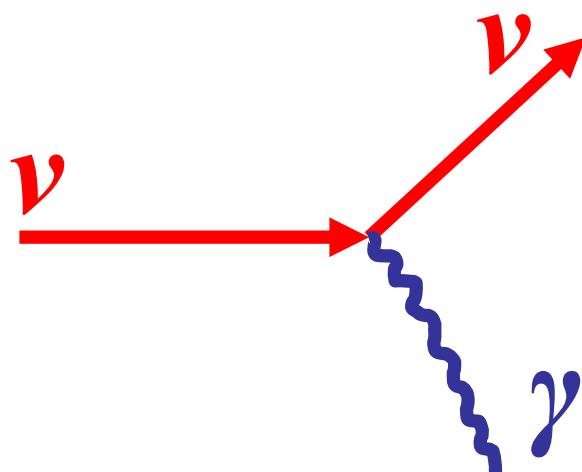
- However, strict requirements for  $Q$  quantization may disappear in extensions of standard  $SU(2)_L \times U(1)_Y$  EW model if  $\nu_R$  with  $Y \neq 0$  are included : in the absence of  $Y$  quantization electric charges  $Q$  gets dequantized

$Q=0$

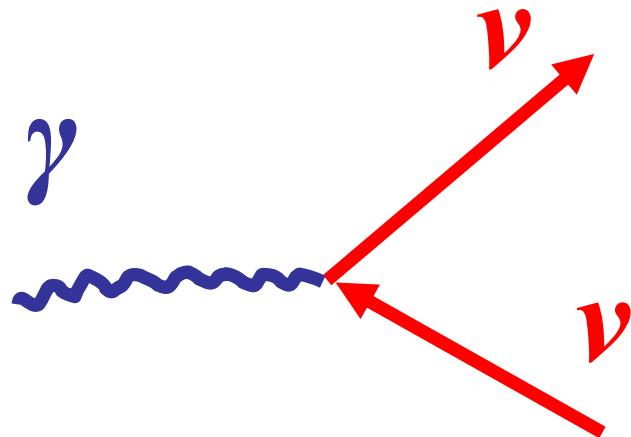
Bardeen, Gastmans, Lautrup, 1972;  
Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000;  
Beg, Marciano, Ruderman, 1978;  
Marciano, Sirlin, 1980; Sakakibara, 1981;  
M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)

millicharged  $\nu$

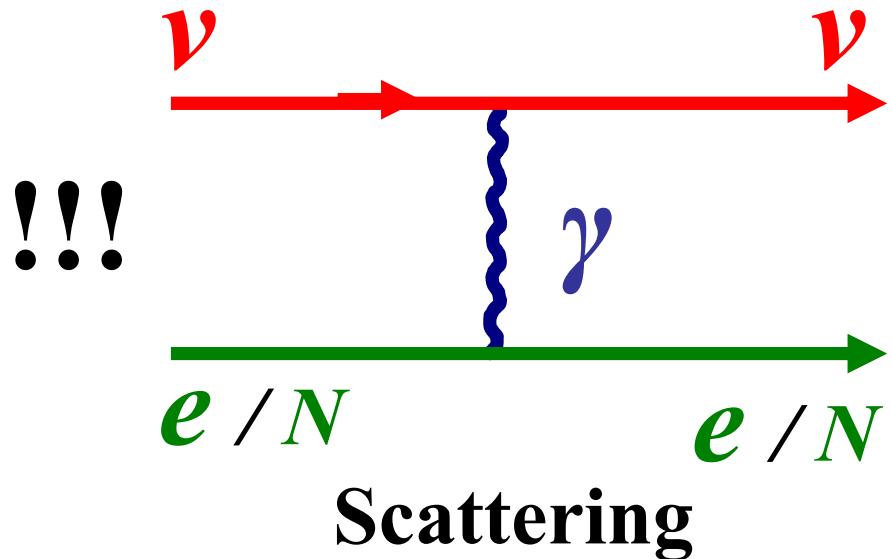
# Neutrino–photon couplings



$\nu$  decay, Cherenkov radiation



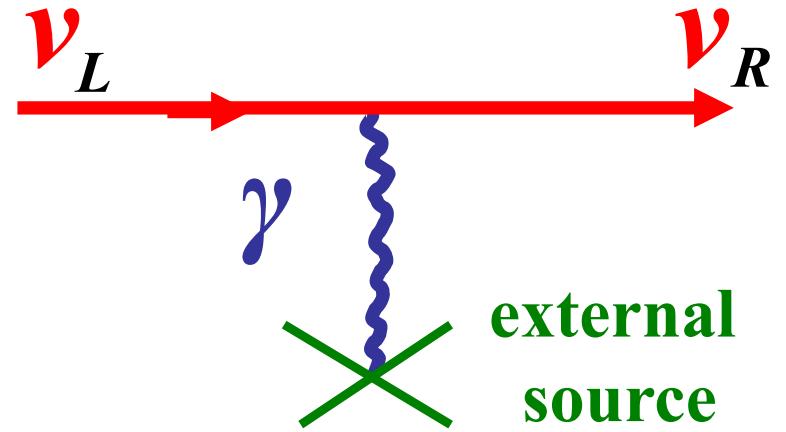
$\gamma$  decay in plasma



!!!

$e/N$

Scattering



Spin precession

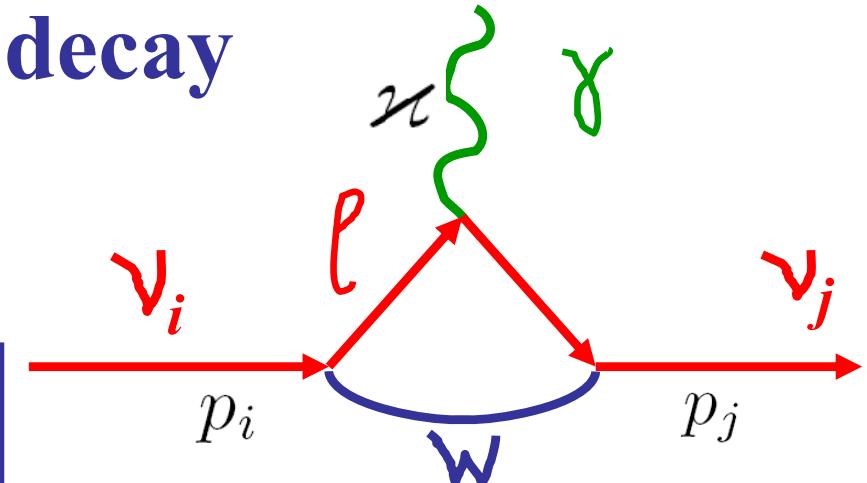
3.7

## Neutrino radiative decay

$$\nu_i \rightarrow \nu_j + \gamma$$

$$m_i > m_j$$

$$L_{int} = \frac{1}{2} \bar{\psi}_i \sigma_{\alpha\beta} (\sigma_{ij} + \epsilon_{ij}\gamma_5) \psi_j F^{\alpha\beta} + h.c.$$



Radiative decay rate

*Petkov 1977; Zatsepin, Smirnov 1978;  
Bilenky, Petkov 1987; Pal, Wolfenstein 1982*

$$\Gamma_{\nu_i \rightarrow \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi} \left( \frac{m_i^2 - m_j^2}{m_i^2} \right)^3 \approx 5 \left( \frac{\mu_{eff}}{\mu_B} \right)^2 \left( \frac{m_i^2 - m_j^2}{m_i^2} \right)^3 \left( \frac{m_i}{1 \text{ eV}} \right)^3 \text{ s}^{-1}$$

$$\mu_{eff}^2 = | \mu_{ij} |^2 + | \epsilon_{ij} |^2$$

- Radiative decay has been constrained from absence of decay photons:

1) reactor  $\bar{\nu}_e$  and solar  $\nu_e$  fluxes,

*Raffelt 1999*

2) SN 1987A  $\nu$  burst (all flavours),

*Kolb, Turner 1990;*

3) spectral distortion of CMBR

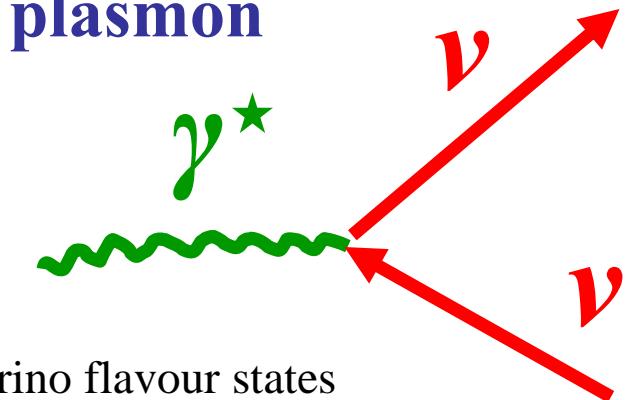
*Ressell, Turner 1990*

### 3.9 The tightest astrophysical bound on $\mu$

G.Raffelt,  
PRL 1990

comes from cooling of red giant stars by plasmon decay  $\gamma^* \rightarrow \nu\bar{\nu}$

$$L_{int} = \frac{1}{2} \sum_{a,b} \left( \mu_{a,b} \bar{\psi}_a \sigma_{\mu\nu} \psi_b + \epsilon_{a,b} \bar{\psi}_a \sigma_{\mu\nu} \gamma_5 \psi_b \right)$$



Matrix element

$$\epsilon_\alpha k^\alpha = 0$$

$$|M|^2 = M_{\alpha\beta} p^\alpha p^\beta, \quad M_{\alpha\beta} = 4\mu^2 (2k_\alpha k_\beta - 2k^2 \epsilon_\alpha^* \epsilon_\beta - k^2 g_{\alpha,\beta}),$$

**Decay rate**

$$\Gamma_{\gamma \rightarrow \nu\bar{\nu}} = \frac{\mu^2}{24\pi} \frac{(\omega^2 - k^2)^2}{\omega}$$

= 0 in vacuum  $\omega = k$

In the classical limit  $\gamma^*$  - like a massive particle with  $\omega^2 - k^2 = \omega_{pl}^2$

**Energy-loss rate per unit volume**

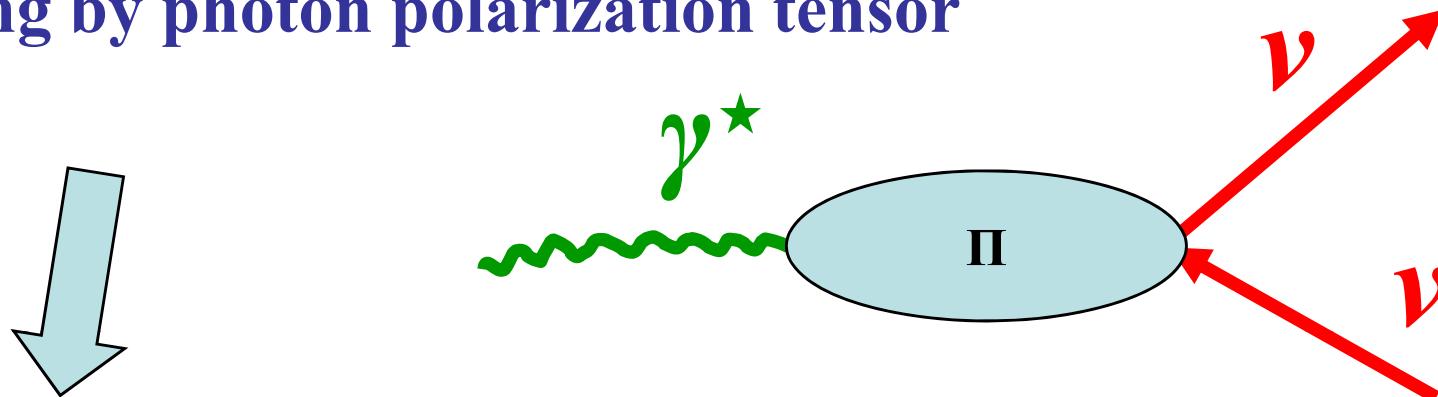
$$\mu^2 \rightarrow \sum_{a,b} (|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2)$$

$$Q_\mu = g \int \frac{d^3 k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \rightarrow \nu\bar{\nu}}$$

distribution function of plasmons

$$Q_\mu = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \rightarrow \nu \bar{\nu}}$$

Magnetic moment plasmon decay  
enhances the Standard Model photo-neutrino  
cooling by photon polarization tensor



more fast cooling of the star.

In order not to delay helium ignition (  $\leq 5\%$  in  $Q$  )



*... best  
astrophysical  
limit on  
magnetic moment...*

$$\mu \leq 3 \times 10^{-12} \mu_B$$

**G.Raffelt,**  
**PRL 1990**

$$\mu^2 \rightarrow \sum_{a,b} \left( |\mu_{a,b}|^2 + |\epsilon_{a,b}|^2 \right)$$

# Astrophysics bounds on $\mu_\nu$

$$\mu_\nu(\text{astro}) < 10^{-10} - 10^{-12} \mu_B$$

Mostly derived from consequences of **helicity-state change** in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Bounds depend on

- modeling of astrophysical systems,
- on assumptions on the neutrino properties.



Generic assumption:

- absence of other nonstandard interactions except for  $\mu_\nu$ .

A global treatment would be desirable, incorporating **oscillation** and **matter effects** as well as the complications due to interference and **competitions among various channels**

Red Giant Lumin.  
 $\mu_\nu \lesssim 3 \cdot 10^{-12} \mu_B$   
G. Raffelt, D. Dearborn,  
J. Silk, 1989.

## 3.10

*Dobroliubov, Ignatiev (1990); Babu, Volkas (1992);  
Mohapatra, Nussinov (1992) ...*

- Constraints on neutrino **millicharge** from **red giants** cooling



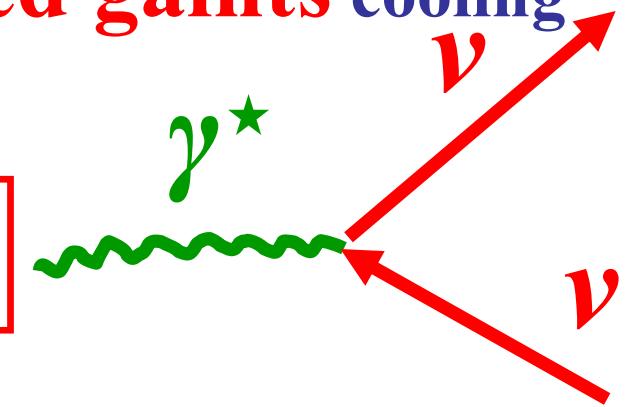
Interaction Lagrangian

$$L_{int} = -iq_\nu \bar{\psi}_\nu \gamma^\mu \psi_\nu A^\mu$$

millicharge

Decay rate

$$\Gamma_{q_\nu} = \frac{q_\nu^2}{12\pi} \omega_{pl} \left( \frac{\omega_{pl}}{\omega} \right)$$



- $q_\nu \leq 2 \times 10^{-14} e$  ...to avoid helium ignition in low-mass **red giants**

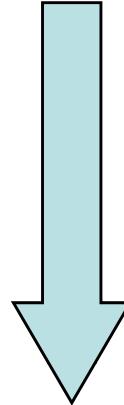
*Halt, Raffelt,  
Weiss, PRL 1994*

- $q_\nu \leq 3 \times 10^{-17} e$  ... absence of anomalous energy-dependent dispersion of SN1987A  $\nu$  signal, most model independent

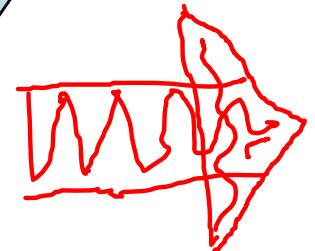
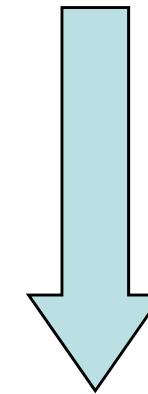
● ... from “charge neutrality” of neutron...

$$q_\nu \leq 3 \times 10^{-21} e$$

# Direct and influence of electromagnetic fields on $\nu$



Indirect



through non-trivial neutrino electromagnetic properties (magnetic moment):

- ★ neutrino spin
- ★ spin-flavour oscillations...
- ★ different  $\nu\gamma$  processes

due to e.m. field influence on charged particles coupled to neutrinos

- ★ neutron beta-decay in  $B$
- ★ change of  $\nu$  oscillation pattern due to matter polarization under influence of external e.m. fields ...

# $\beta$ -decay of neutron in magnetic field

{Birth of  $\gamma$  astrophysics in B}



- \* L. Korovina, "Beta-decay of polarized neutron in magnetic field", Sov.Phys.J., # 6 (1964) 86
- \* I.Ternov, B.Lysov, L.Korovina, Mosc.Univ.Bull.,Phys.,Astron., #5 (1965) 58  
"On the theory of neutron  $\beta$ -decay in external magnetic field."
- \* J.Matese, R.O'Connell, "Neutron beta decay in a uniform magnetic field", Phys.Rev.180 (1969) 1289
- \* L.Fassio-Canuto, "Neutron beta decay in a strong magnetic field" Phys.Rev.187 (1969) 2141
- \* G.Greenstein, Nature 223 (1969) 938

## \* Asymmetry in $\tilde{\nu}$ emission

$$\frac{W(B)}{W_0} = \frac{1}{2} \int \sin \theta d\theta \left\{ 1 + \frac{2(\alpha^2 - \alpha)}{1+3\alpha^2} S_n \underline{\cos \theta} \right.$$

$$- 4.9 \frac{eB}{\Delta^2} \left( \frac{\alpha^2 - 1}{1+3\alpha^2} \underline{\cos \theta} + \frac{2(\alpha^2 + \alpha)}{1+3\alpha^2} S_n \right) \}$$



astrophysical  
applications



K.Kouzakov, A.Studenikin  
Phys.Rev.C 72 (2005) 015502



“Bound-state beta-decay  
of neutron in strong  
magnetic field”

Usual (continuum - state)  $\beta$  decay     $n \rightarrow p + e^- + \bar{\nu}_e$   
"Rare" (bound - state)  $\beta$  decay     $n \rightarrow (pe^-) + \bar{\nu}_e$

R. Daudel, M. Jean, and M. Lecoin, J. Phys. Radium **8**, 238 (1947)

$$\frac{w_b}{w_c} \simeq 4.2 \times 10^{-6}$$

$$\tau_c \sim 15 \text{ min}$$
  
$$\tau_b \sim 7 \text{ years}$$

J.N. Bahcall, Phys. Rev. **124**, 495 (1961) [Dirac equation]

L.L. Nemenov, Sov. J. Nucl. Phys. **15**, 582 (1972) [Schrödinger equation]

X. Song, J. Phys. G: Nucl. Phys. **13**, 1023 (1987) [Bethe-Salpeter equation]

## Summary

First analysis of bound-state  $\beta$  decay in a strong magnetic field ( $B \sim 10^{13}$ - $10^{18}$  G)

- ✓  $w_b/w_c \sim 0.1$ - $0.4$  in contrast to the field-free case, where  $w_b/w_c \sim 10^{-6}$
- ✓ A logarithmiclike behavior  
 $w_b/w_c \propto \log_{10}(B/B_e) + b$  ( $b > 0$ )

**Outlook:** Astrophysical applications?

3.12

✓ e.m. form factors are affected by matter and  $B$



magnetic moment  $\mu_\nu = \mu_\nu(B)$



induced electric charge of  $\nu$   
in magnetized matter

Egorov  
Studenikin  
1992

Borisov,  
Zhukovsky,  
Kurilin,  
Ternov,  
1985



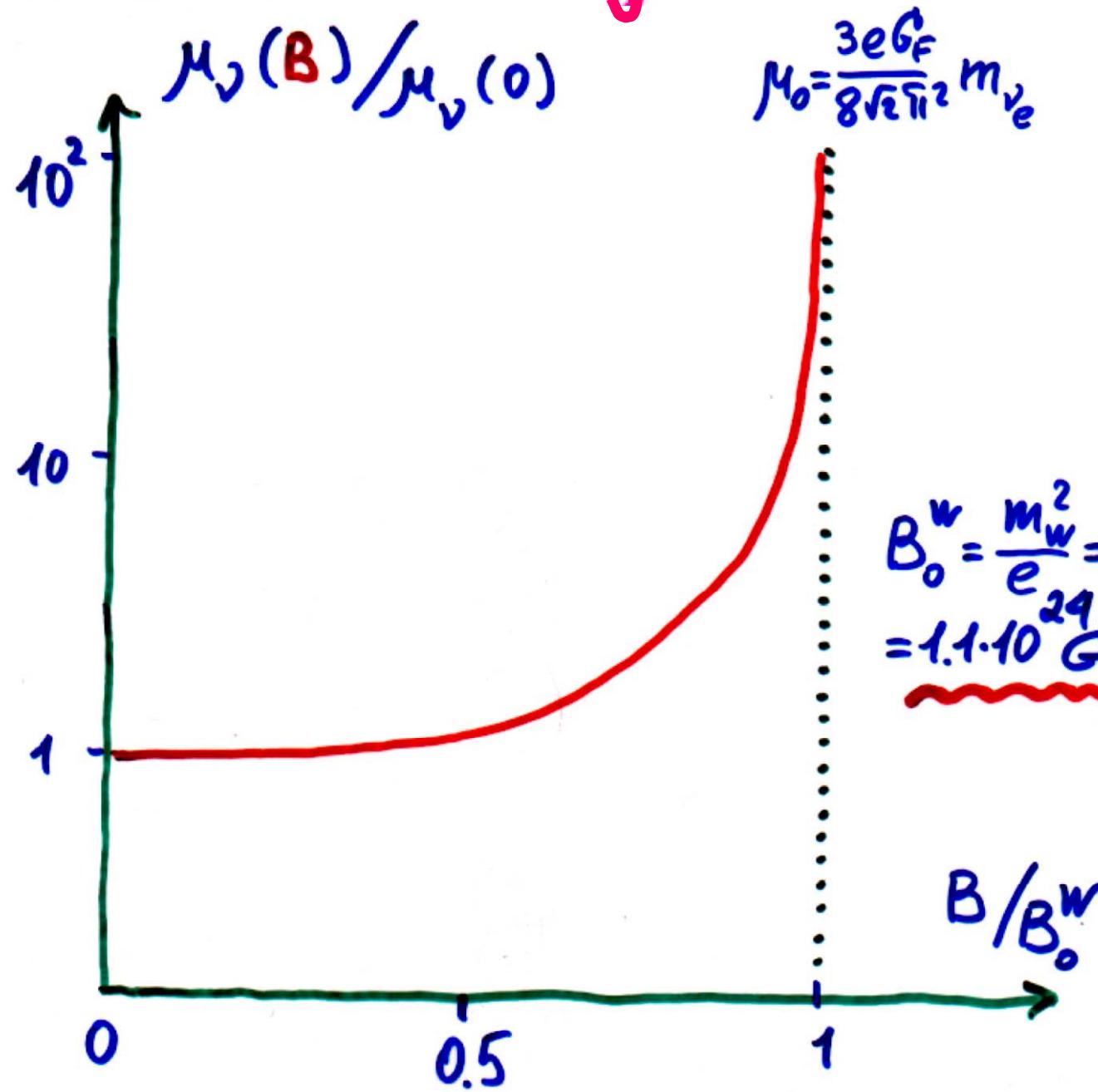
Oraevsky, Semikoz

Smorodinsky, 1986

Bhattacharaya, Ganguly, Konar, 2002

Nieves, 2003

# Neutrino magnetic moment



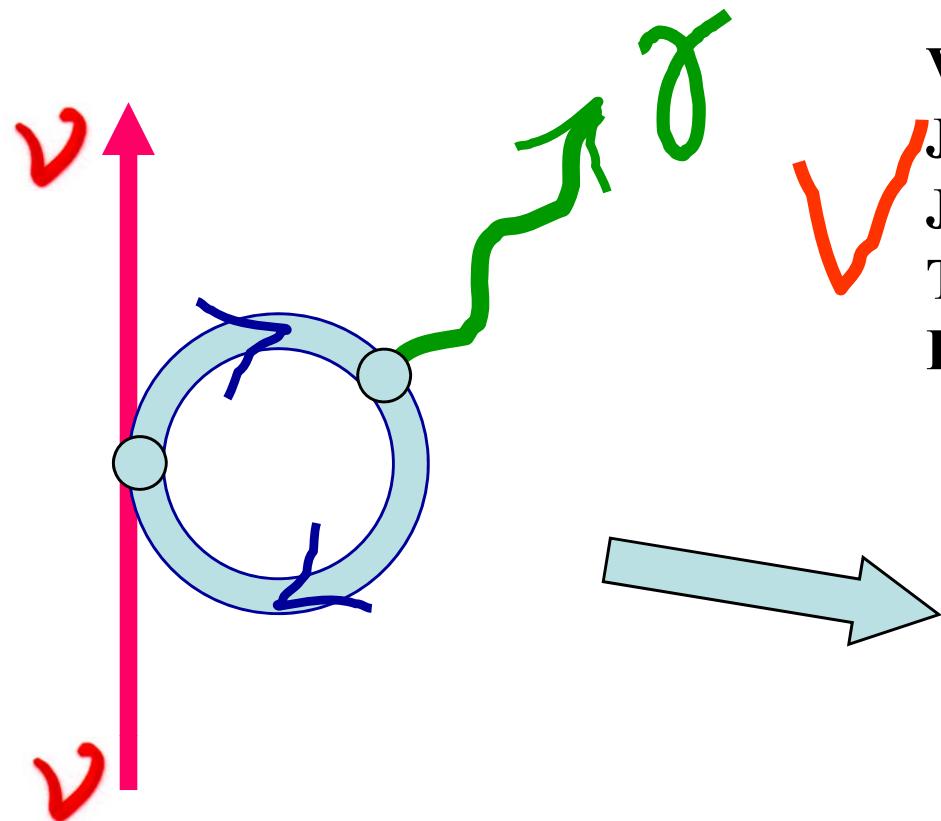
Borisov,  
Zhukovskiy,  
Kurilin,  
Ternov, 1985 ;

Masood,  
Perez Rojas,  
Gaitan,  
Rodrigues-Romo,  
1999

$\nu$

# “effective electric charge” in magnetized plasma

- $\nu$ s do not couple with  $\gamma$ s in vacuum,  
... however, when
- $\nu$  in thermal medium ( $e^-$  and  $e^+$ )



V.Oraevsky, V.Semikoz, Ya.Smorodinsky,  
JETP Lett. 43 (1986) 709;  
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;  
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;  
K.Bhattacharya, A.Ganguly, 2002

...different  $\nu\gamma$  interactions in  
astrophysical and cosmological media

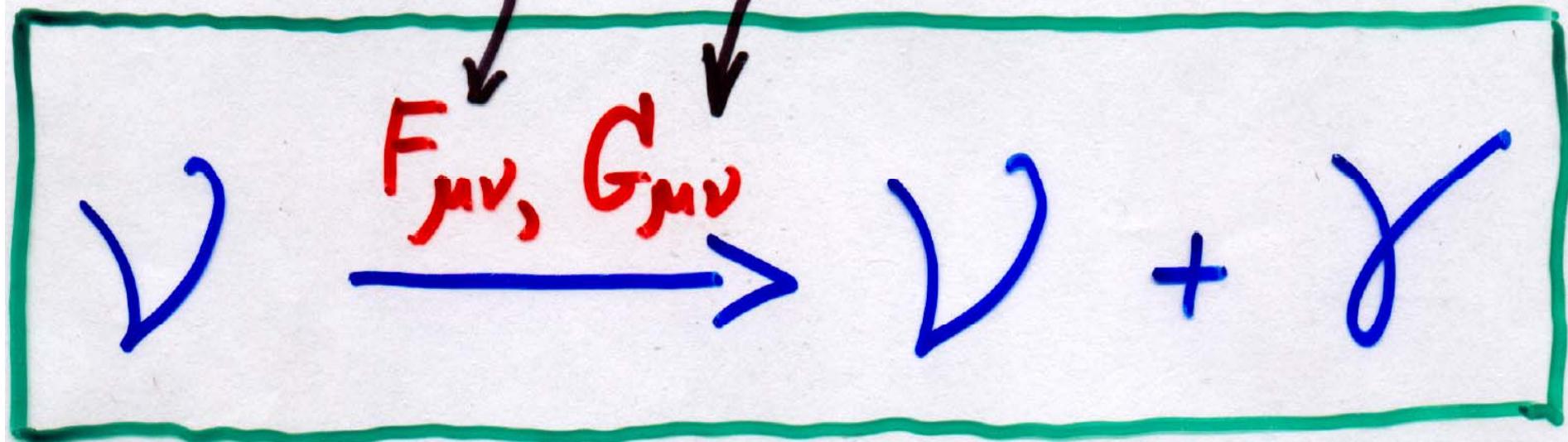
# New mechanism of electromagnetic radiation

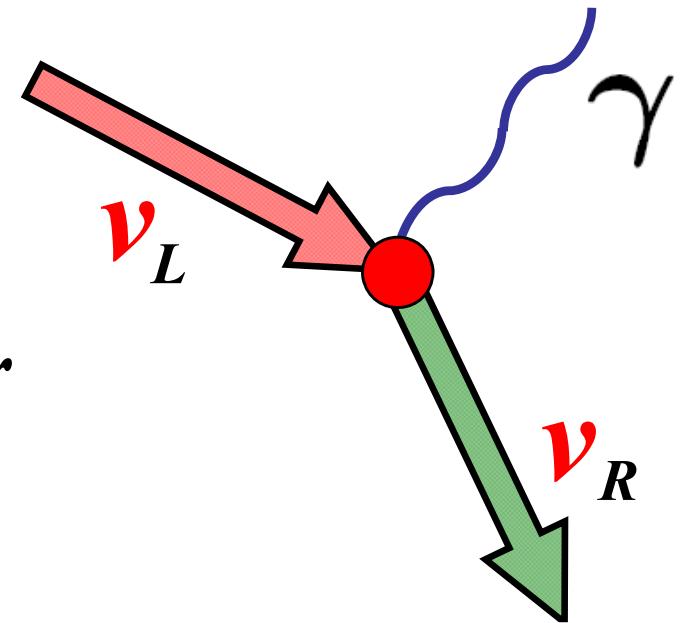
"Spin light of neutrino"

in matter and

electromagnetic fields

SL $\nu$





## *Spin light of neutrino in matter*

● new mechanism of the electromagnetic process  
stimulated by the presence of matter, in which  
neutrino with non-zero magnetic moment emits light

*A.Lobanov, A.Studenikin, Phys.Lett. B 564 (2003) 27,  
Phys.Lett. B 601 (2004) 171*

*A.S., A.Ternov, Phys.Lett. B 608 (2005) 107*

*A.Grigoriev, A.S., A.Ternov, Phys.Lett. B 622 (2005) 199*

*A.S., J.Phys.A: Math.Gen. 39 (2006) 6769*

*A.S., J.Phys.A: Math.Theor. 41 (2008) 16402*

# New mechanism of electromagnetic radiation

? Why Spin Light  
of neutrino       $SL\nu$   
of electron       $SLe$       in matter.

Analogies with :

\* classical electrodynamics

an object with charge  $Q = 0$  and

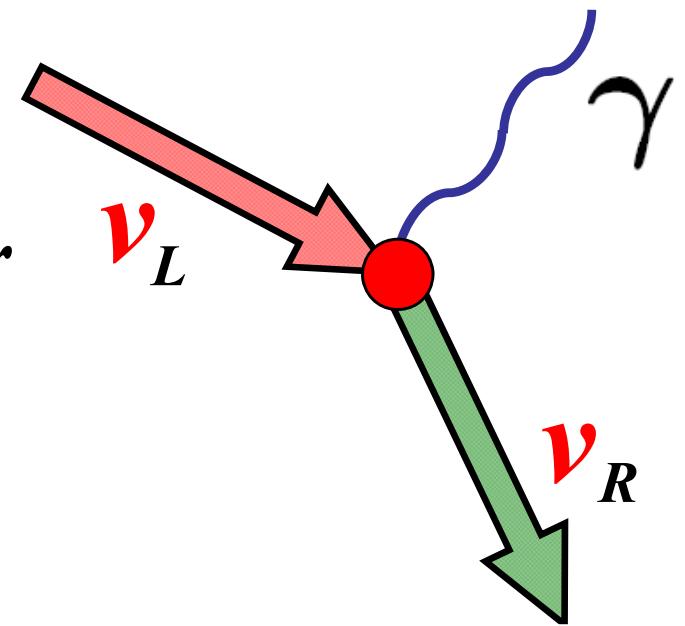
magnetic moment  $\vec{m} = \frac{1}{2} \sum_i e_i [\vec{r}_i \times \vec{v}_i] \neq 0$

$$I^{\text{cl. el.}} = \frac{2}{3} \vec{m}^2$$

magnetic dipole  
radiation power

## *Spin light of neutrino in matter*

*(quantum approach)*



- new mechanism of the electromagnetic process stimulated by the presence of matter, in which a neutrino with non-zero magnetic moment emits light.

*A.Studenikin, A.Ternov, PLB 2005*

*A.Grigoriev, Studenikin, Ternov, PLB 2005*

*A.S., J.Phys.A: Math.Gen. 39 (2006) 6769*

*A.S., J.Phys.A: Math.Theor. 41 (2008) 16402*

# Modified Dirac equation for neutrino in matter

Addition to the vacuum neutrino Lagrangian

$$\Delta L_{eff} = \Delta L_{eff}^{CC} + \Delta L_{eff}^{NC} = -f^\mu \left( \bar{\nu} \gamma_\mu \frac{1 + \gamma^5}{2} \nu \right)$$

matter current

where

$$f^\mu = \frac{G_F}{\sqrt{2}} \left( (1 + 4 \sin^2 \theta_W) j^\mu - \lambda^\mu \right)$$

matter polarization

$$\left\{ i \gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

It is supposed that there is a macroscopic amount of electrons in the scale of a neutrino de Broglie wave length. Therefore, **the interaction of a neutrino with the matter (electrons) is coherent.**

L.Chang, R.Zia,'88; J.Panteleone,'91; K.Kiers, N.Weiss,  
M.Tytgat,'97-'98; P.Manheim,'88; D.Nötzold, G.Raffelt,'88;  
J.Nieves,'89; V.Oraevsky, V.Semikoz, Ya.Smorodinsky,89;  
W.Naxton, W-M.Zhang'91; M.Kachelriess,'98;  
A.Kusenko, M.Postma,'02.

A.Studenikin, A.Ternov, hep-ph/0410297;  
*Phys.Lett.B 608 (2005) 107*

This is the most general equation of motion of a neutrino in which the effective potential accounts for both the **charged and neutral-current** interactions with the background matter and also for the possible effects of the matter **motion and polarization**.

# Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter

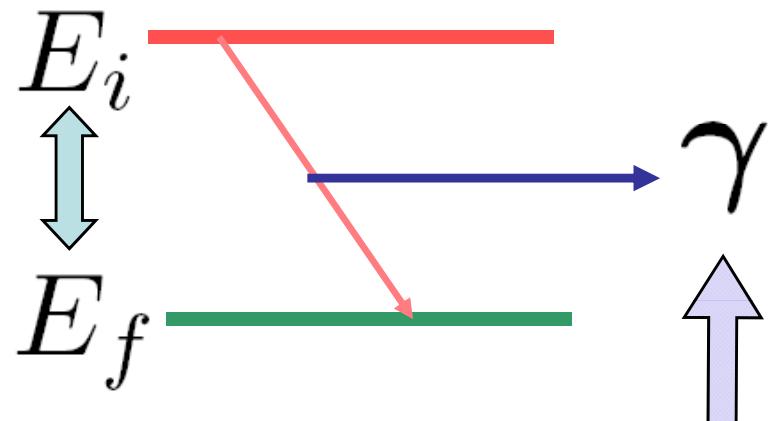
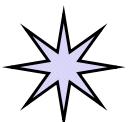
shows that this process originates from the **two subdivided phenomena**:



the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,

$$E = \sqrt{p^2 \left(1 - s\alpha \frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$



the radiation of the photon in the process of the neutrino transition from the **“excited” helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov,

A.Grigoriev, A.Studenikin, A.Ternov,

Phys.Lett.B 608 (2005) 107;

Phys.Lett.B 622 (2005) 199;

Grav. & Cosm. 14 (2005) 132;

**neutrino-spin self-polarization effect in the matter**

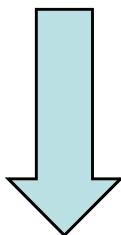
A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;  
Phys.Lett.B 601 (2004) 171

# Method of exact solutions

Modified **Dirac equations** for  $\nu$  (and  $e$ )  
(containing the correspondent effective matter potentials)



**exact solutions** (particles wave functions)



a basis for investigation of different phenomena which  
can proceed when **neutrinos** and **electrons** move in  
dense media  
**(astrophysical** and **cosmological** environments).

«method of exact solutions »

# Interaction of particles in external electromagnetic fields

( Furry representation in quantum electrodynamics )

Potential of electromagnetic field

$$A_\mu(x) = A_\mu^q(x) + A_\mu^{ext}(x),$$

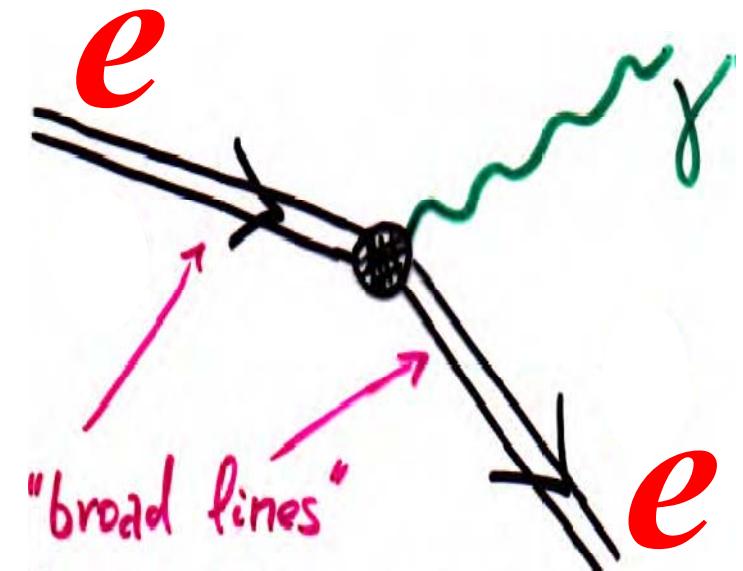
evolution operator

$$U_F(t_1, t_2) = T \exp \left[ -i \int_{t_1}^{t_2} j^\mu(x) A_\mu^q(x) dx \right],$$

charged particles current

$$j_\mu(x) = \frac{e}{2} [\Psi_F \gamma_\mu, \Psi_F],$$

$e \xrightarrow{\text{B}_\perp} e + \gamma$   
synchrotron radiation

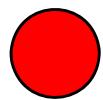


Dirac equation in external classical (non-quantized) field  $A_\mu^{ext}(x)$

$$\left\{ \gamma^\mu \left( i \partial_\mu - e A_\mu^{ext}(x) \right) - m_e \right\} \Psi_F(x) = 0$$

- ...beyond perturbation series expansion,  
strong fields and non linear effects...

## ... evaluation of the method



- within a project of research of  
in dense matter and external fields
- stimulated by a need to obtain a consistent  
theory of “spin light of neutrino (electron)  
in matter”



A.Studenikin,

“Neutrinos and electrons in background matter: a new approach”,  
**Ann.Fond. de Broglie 31 (2006) 289;**

“Method of wave equations exact solutions in studies of neutrino and  
electron interactions in dense matter”,  
**J.Phys.A: Math.Theor. 41 (2008) 164047**

*QFEXT’07*

$\nu$  and  $e$

in matter being treated within  
the method of exact solutions  
of quantum wave equations -

**«method of exact solutions »**

A.Studenikin, A.Ternov,  
**Phys.Lett.B 608** (2005) 107;

**hep-ph/0410297**,  
“Neutrino quantum states in matter”;

**hep-ph/0410296**,  
“Generalized Dirac-Pauli equation  
and neutrino quantum states in  
matter”

A.Grigoriev, A.Studenikin,  
A.Ternov,  
**Phys.Lett.B 608 622** (2005) 199

A.Studenikin,

**J.Phys.A: Math.Theor.** **41** (2008) 16402,  
“Method of wave equations exact solutions  
in studies of neutrino and electron  
interactions in dense matter”

**Ann. Fond. de Broglie** **31** (2006) 289,  
“Neutrinos and electrons in background  
matter: a new approach”

**J.Phys.A: Math.Gen.** **39** (2006) 6769

# Neutrino energy quantization in rotating media: new mechanism for neutrino trapping inside dense rotating stars

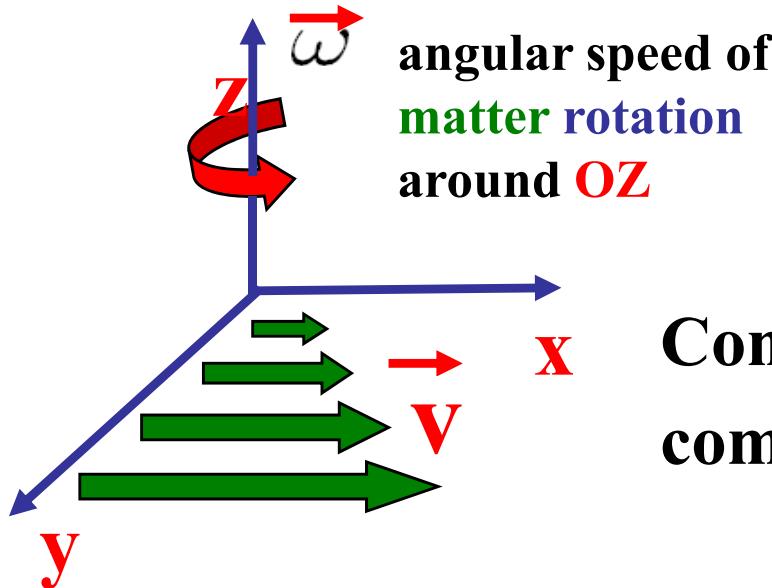
**Neutrino'08,  
Christchurch,  
May 25-31, 2008**

**Alexander Studenikin**

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**A.Studenikin, “Method of exact solutions in  
studies of neutrinos and electrons in dense matter”  
J.Phys.A:Math.Theor. 41 (2008) 164047 (20 pp)**

# Neutrino energy quantization in matter



angular speed of  
matter rotation  
around OZ

Consider  $\nu$  moving in **rotating** medium  
composed of neutrons (generalization s.f.):

$\nu$  wave function

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

where **matter potential**  $f^\mu = -G(n, n\mathbf{v})$ ,  $\mathbf{v} = (\omega y, 0, 0)$ ,  $\rho = Gn\omega$   $G = \frac{G_F}{\sqrt{2}}$   
**neutron number density**

speed of matter      angular speed of rotation

$\nu$  energy spectrum

$$\tilde{p}_0 = \sqrt{p_3^2 + 2\rho N} + Gn, \quad N = 0, 1, 2, \dots$$

→ circular orbits → trapping inside dense stars

*A. Grigoriev, A. Savochkin, A. Studenikin,  
Russ.Phys.J. 50 (2007) 845  
A. Studenikin,  
J.Phys.A:Math.Theor. 41 (2008) 164047*

*A. Ternov,  
A. Studenikin,  
Phys.Lett.B 608 (2005) 107;  
A. Grigoriev, A.S., A. Ternov,  
Phys.Lett.B 622 (2005) 199*

... consistent model of a rotating matter with account for  $\nu$  mass

*I.Balantsev, Yu.Popov, A.Studenikin,  
Nuov.Cim.B 32 (2009) 53,  
arXiv: 0906.2391*

$$\left\{ i\gamma_\mu \partial^\mu - \frac{1}{2} \gamma_\mu (1 + \gamma_5) f^\mu - m \right\} \Psi(x) = 0$$

$$f^\mu = -G(n, n\mathbf{v}), \quad \mathbf{v} = (-\omega y, \omega x, 0)$$

## Energy spectra

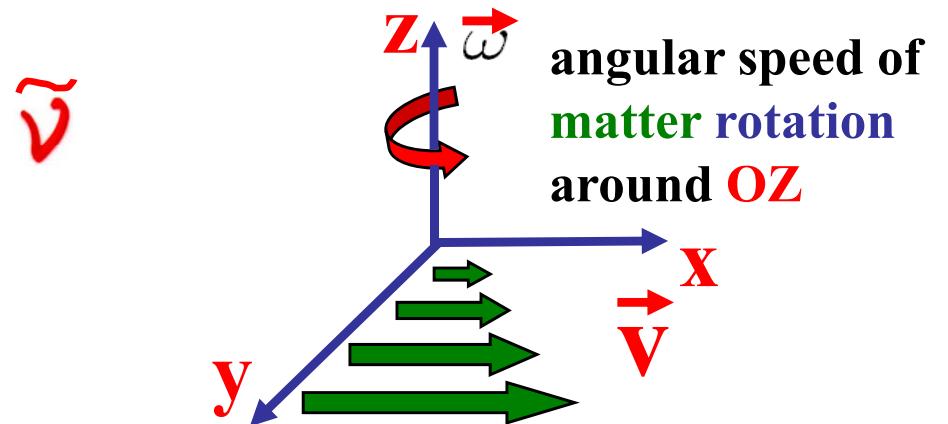
$$p_0 = \sqrt{m^2 + p_3^2 + 4N\rho} - Gn \quad \text{for } \nu$$

$$\tilde{p}_0 = \sqrt{m^2 + p_3^2 + 4N\rho} + Gn \quad \text{for } \tilde{\nu}$$

$$N = 0, 1, 2, \dots \quad \rho = Gn\omega$$

**One example:** consider antineutrino in rotating neutron matter, then energy of transversal motion

$$\tilde{p}_\perp = \sqrt{2\rho N} \quad \rho = G n \omega$$



Quantum number  $N$  also determines **radius** of antineutrino quasi-classical orbit in moving matter:

$$R = \sqrt{\frac{2N}{Gn\omega}}$$

binding orbits inside a Neutron Star !?

NS:

$$R_{NS} = 10 \text{ km}$$

$$n = 10^{37} \text{ cm}^{-3}$$

$$\omega = 2\pi \times 10^3 \text{ s}^{-1}$$

for this set

radius of trajectory

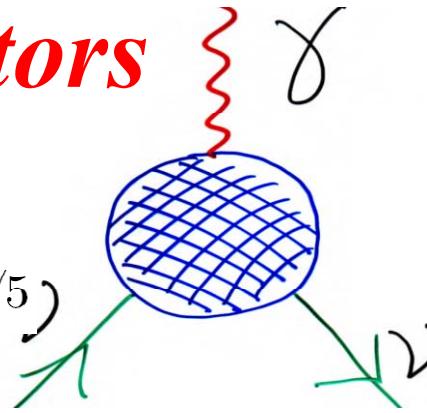
$$R = \sqrt{\frac{2N}{Gn\omega}} \quad R_{NS} = 10 \text{ km}$$

if  $N \leq N_{max} = 10^{10}$

with  $N \leq 10^{10}$  can be bound inside the star

thus,  $\tilde{\nu}$  with energy  $\tilde{p}_0 \sim 1 \text{ eV}$  can be bound inside NS  
 $N \gg 1$  and  $p_3 = 0$

# *Conclusion*

- V e.m. vertex function  $\rightarrow$  4 form factors**
- 
- charge      dipole magnetic and electric
- $\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu\cancel{q})\gamma_5$  **anapole**
  - EM properties  $\rightarrow$  a way to distinguish **Dirac** and **Majorana** **V**

- Standard Model with  $\nu_R$  ( $m_\nu \neq 0$ ):  $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_\nu \sim 3 \cdot 10^{-19} \mu_B \left(\frac{m_\nu}{1 \text{ eV}}\right)$
- In extensions of SM  $\rightarrow$ 
  - enhancement of magnetic moment **V**, even
  - electrically millicharged **V**
- Limits from reactor **v-e** scattering experiments (2010):
 

$\mu_\nu < 3.2 \times 10^{-11} \mu_B$

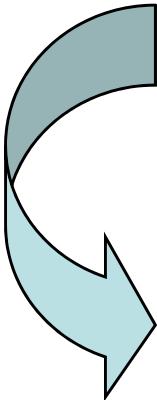
*A.Beda et al.  
(GEMMA Coll.)*
- Limits from astrophysics, star cooling (1990):
 

$\mu \leq 3 \times 10^{-12} \mu_B$

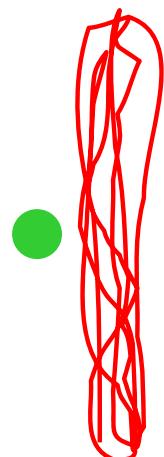
*G.Raffelt*

$\mu_{\nu}$  is presently known to be in the range

$$10^{-20} \mu_B \leq \mu_{\nu} \leq 10^{-11} \mu_B$$



$\mu_{\nu}$  provides a tool for exploration possible physics beyond the Standard Model

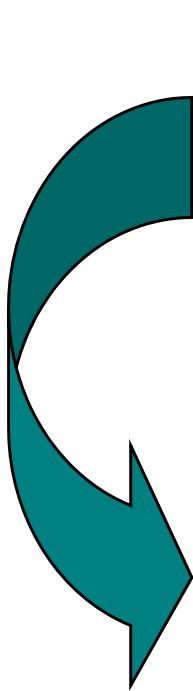


Due to smallness of neutrino-mass-induced magnetic moments,

$$\mu_{ii} \approx 3.2 \times 10^{-19} \left( \frac{m_i}{1 \text{ eV}} \right) \mu_B$$

any indication for non-trivial electromagnetic properties of  $\nu$ , that could be obtained within reasonable time in the future, would give evidence for interactions beyond extended Standard Model

*... situation with*



V

electromagnetic properties

*is better than it was for V  
in the time of w. Pauli, 1930*

*... once they will be observed experimentally*

*... are important in astrophysics*

*... there is a need for further theoretical and  
experimental studies*

